Trust and Social Collateral*

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Abstract

This paper builds a theory of informal contract enforcement in social networks. In our model, relationships between individuals generate social collateral that can be used to control moral hazard when agents interact in a borrowing relationship. We define trust between two agents as the maximum amount that one can borrow from the other, and derive a simple reduced form expression for trust as a function of the social network. We show that trust is higher in more connected and more homogenous societies, and relate our trust measure to commonly used network statistics. Our model predicts that dense networks generate greater welfare when arrangements typically require high trust, and loose networks create more welfare otherwise. Using data on social networks and behavior in dictator games, we document evidence consistent with the quantitative predictions of the model.

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Lending a valuable asset such as money or a car usually exposes the lender to moral hazard, but can also create substantial social surplus. To realize this surplus, many developed economies rely on formal contracts enforced by the legal system. For example, in most OECD countries renting a car is a simple process involving a temporary charge on one's credit card. However, as Dixit (2004) reminds us, low-cost legal contract enforcement is historically a relatively new phenomenon and still absent in most developing countries: "In all countries through much of their history, the apparatus of state law was very costly, slow, unreliable, biased, weak, or simply absent. In most countries this situation still prevails."

When legal contract enforcement is unavailable or costly, economic agents turn to informal arrangements. As we document in Section 1, such arrangements often rely on social networks which facilitate trust between contracting parties. For example, Singerman (1995) shows that among the lower class of Cairo, knowing the leader of an informal savings association (called the gam'iyyaat) is generally necessary to obtain a loan; in fact, social links are often deliberately built through marriage to maximize access to resources. The social network can be used to secure borrowing in advanced countries as well: the best way to find temporary accommodation in Paris might be to have a friend or a friend of a friend who owns an apartment there.

These observations suggest that the structure of the social network can be an important determinant of trust and economic outcomes. Our paper provides theoretical support for this view by developing a model of informal contract enforcement in the social network. In our model, direct relationships between agents generate value which can be used as *social collateral* to facilitate informal borrowing arrangements. This collateral role of the social network can be interpreted as one aspect of social capital (Coleman 1988, Putnam 2000). We define *trust* between two agents as the amount that they can borrow from each other, and derive a simple reduced form expression for trust as a function of the network structure. We then use this reduced form to explain a number of stylized facts about trust and social capital and to derive additional empirical implications.

The basic connection between the social network and enforceable borrowing arrangements can be seen with the examples in Figure 1. In these examples, agent s would like to borrow an asset, like a car, from agent t, in an economy with no centralized means of contract enforcement. Begin with Figure 1a, where the network consists of three agents: s, t and their common friend u. Let the value of the friendship between s and u be 3, and that between u and t be 4. In our model, t will lend the car only if its value does not exceed $\min[3, 4] = 3$, the value of the weakest link on the path connecting s and t. To see the logic, imagine that s chooses not to return the car. Then u will threaten to stop providing friendship services to s. This threat can enforce a payment of up to 3, the value of the connection between s and u. For t to receive this payment, he must have enough leverage over u as measured by the value of their friendship, explaining the role of the weakest link in determining the borrowing limit. Now consider Figure 1b, where agents s and t have a second common friend v. Let the value of the friendship between s and v be 2, and that between v and t be 1. Here, the borrowing limit increases to $\min[2,1] + \min[3,4] = 4$, where $\min[2,1] = 1$ is the weakest link on the second path between s and t. Trust is now higher because both intermediate agents can vouch for part of the value of the car.

Our main theoretical result is that in general networks, the level of trust equals the sum of the weakest link values over all disjoint paths connecting borrower and lender. This quantity is called the *maximum network flow*, a well-studied concept in graph theory.[1] Intuitively, the maximum flow is the highest amount that can flow from borrower to lender along the edges of the network respecting the capacity constraints given by link values. The proof of our main result builds on the maximum flow-minimum cut theorem (Ford and Fulkerson 1956), a famous characterization result for network flows. We show that network flows can be used to characterize borrowing in two more general environments as well: in economies with multiple borrowers and lenders, and in the presence of cash- or time-constraints on agents' ability to make payments.

The characterization of trust in terms of network flows provides a simple and intuitive reduced form which we then use in three applications. First we explore the comparative statics of trust with respect to changes in network structure. We find that increasing the number or strength of links provides greater leverage between unconnected agents and hence leads to more trust, supporting Putnam's (1995) argument that "networks of civic engagement (...) encourage the emergence of social trust." We also show that greater heterogeneity in link strength leads to lower average trust. This is because trust is determined by the value of the weakest link along certain paths, which is on average reduced when heterogeneity increases. Our model predicts that societies with greater ethnic or racial heterogeneity should exhibit lower trust, consistent with empirical findings by Alesina and La Ferrara (2002) and others.

As a second application we explore the connection between our trust measures and standard network statistics. When we restrict attention to informal arrangements that only involve agents who are not too distant from the borrower in the social network, our trust measures are functions of

[1] See Cormen, Leiserson, Rivest, and Stein (2001) for a textbook treatment.

three common network statistics:[2] 1) the number of friends of an agent; 2) the clustering coefficient of an agent, which is a measure of local network density; 3) the number of common friends of two agents. It follows that our trust measures can be computed in many cases even with limited network data, increasing their potential applicability in empirical work. In addition, our results can be viewed as microfoundations for common network statistics in terms of trust and social collateral.

In a third application, we use our model to address a debate in sociology about the relative advantage of different network structures. Coleman (1988) stresses the importance of dense networks, referred to as network closure, which can facilitate the enforcement of cooperation. In contrast, Burt (1995) emphasizes structural holes, i.e., agents who bridge otherwise disconnected networks, and argues and that loose networks are better because they provide greater access to information and other resources. In our model, the relative benefit of these network structures depends on the type of assets borrowed. Closure is more attractive when the assets borrowed are highly valuable, because closure maximizes the level of trust among a small number of individuals. This is in line with Coleman's example of diamond dealers in New York City, who exchange highly valuable stones and form a tight network of friendship, family, and religious ties. In contrast, when the network is mainly used to exchange assets of small value like information, large and loose neighborhoods are better because they maximize the probability of access to these assets.

At the end of the paper, we present evidence on the relationship between network flow measures of trust and trusting behavior in practice. We use experimental and survey data collected from Harvard undergraduates, and compute flow measures of trust using self-reported data on their social network. Adopting the approach of Glaeser, Laibson, Scheinkman, and Soutter (2000), we measure trusting behavior by observing subjects' play in dictator game experiments. Our network-based measures of trust are strongly correlated with trusting behavior in dictator games, even after controlling for demographics and various proxies for social distance. These results suggest that our measures of trust can be of additional use in applied work.

This paper builds a theory of trust that arises from contract enforcement. As Karlan, Mobius, and Rosenblat (2006) discuss, other mechanisms can also generate trust between socially close agents. For example, a lender might feel more altruistic towards, or know more about the type (e.g., reliability), of a friend than a stranger. While these mechanisms are important sources of trust in practice, we abstract away from them to better focus on trust arising from contract enforcement.

Our model is related to the literature on cooperation in repeated interactions in the absence of

[2] See e.g., Watts and Strogatz (1998), Glaeser, Laibson, Scheinkman, and Soutter (2000) and Jackson (2005).

formal institutions. Kandori (1992), Greif (1993) and Ellison (1994) develop models of community enforcement where deviators are punished by all members of society. More related is Dixit (2003), who studies cooperation with local enforcement among agents in a fixed circle network; in contrast, we explore the effect of different network structures. In related work, Bloch, Genicot, and Ray (2005) build a model of informal insurance in social networks. Our paper also builds on research in economics on social capital and the measurement of trust. Glaeser, Laibson, and Sacerdote (2002) construct an economic model of social capital but do not study networks. Glaeser, Laibson, Scheinkman, and Soutter (2000) show using trust game experiments that trust is higher between agents who are closer socially.

The rest of the paper is organized as follows. Section 1 collects motivating evidence on the role of networks in informal arrangements. Section 2 develops the model and derives the reduced form expression for trust. A reader more interested in applications might wish to skip ahead to Section 3, where we derive comparative statics, compute trust from standard network statistics, and address the debate on closure versus structural holes. Section 4 presents new evidence about social collateral and trusting behavior, and Section 5 concludes. All proofs are given in the Appendix.

1 Social networks and informal arrangements: Evidence

This section presents evidence that people rely on their social network in a variety of exchanges. In many of these exchanges, the social network provides two related services: (1) access to the asset that is to be exchanged; (2) trust between the parties. The mechanism by which the social network performs these services can be illustrated using an example originally due to Wechsberg (1966), which we take from Coleman (1990). This example is about a prominent Norwegian shipowner who was in need of a ship which had undergone repairs in an Amsterdam shipyard. However, "the yard would not release the ship unless a cash payment was made of 200,000 pounds. Otherwise the ship would be tied up for the weekend, and the owner would lose at least twenty thousand pounds." The shipowner was in trouble, because he did not have access to 200,000 pounds to be delivered immediately in Amsterdam. To solve this problem, he called a London banker at Hambros, who presumably had contacts in Amsterdam. After hearing the situation, "the Hambros man looked at the clock and said, 'It's getting late but I'll see whether I can catch anyone at the bank in Amsterdam ... stay at the phone.' Over a second phone he dictated to a secretary in the bank a telex message to the Amsterdam bank: "Please pay 200,000 pounds telephonically to (name) shipyard on understanding that (name of ship) will be released at once."

In this example, the shipowner borrowed 200,000 pounds on immediate notice from an Amsterdam bank that he had no direct connection with. He accomplished this using two business relations: his connection with the London banker, and the connection between the London and Amsterdam banks. In Coleman's (1990) terminology, the London banker acted as a "trust intermediary": he provided access and created trust between two individuals who did not know each other. If the shipowner were to default, the Amsterdam bank could ask the London banker to pay compensation or risk jeopardizing their relationship. Similarly, the London banker could presumably extract money from the shipowner if necessary. Thus, the two business relations were used as collateral to secure borrowing.

More generally, Table 1 documents evidence, drawn from a number of sources, that people use their social network in many informal exchanges. In each row we report the proportion of individuals who would or do rely on their social network in a particular exchange. Panel A focuses on borrowing. According to the first row, 55% of subjects in the 1995 General Social Survey (GSS), when asked whom they would turn to if they needed to borrow a large sum of money, chose persons who are in their direct social network, including family members, relatives and close friends.[3] The next two rows show that in less developed countries, borrowing by small enterprises also frequently relies on social networks.[4]

Panel B documents evidence of network use in the purchases of valuable assets. For such purchases, particularly when there is asymmetric information about the good, trust between buyer and seller can be important for a successful transaction. As the table reports, in 40% of home purchases by subjects in the 1996 GSS, there is a direct or indirect network connection between the buyer and either the seller or the realtor. Similarly, 44% of used car purchases involve a direct or indirect network connection between the buyer and the seller. Finally, Panel C shows that networks play an important role in job search: e.g., data from the 1991 and 1992 Current Population Surveys indicates that 23% of unemployed workers used friends and relatives to search for jobs.[5]

Taken together, this evidence suggests that social networks matter for facilitating arrangements and generating trust. We now develop a model where networks build trust through social collateral,

[3] See the data appendix for question wording and other details for all environments.

[4] Indirect evidence on the use of social networks for borrowing, gifts, and transfers can be found in the literature on consumption smoothing in developing countries, including Townsend (1994), Fafchamps and Lund (2003), Angelucci and De Giorgi (2006) and Karlan (2006).

[5] In job referrals, trust can play a role to the extent that the referring agent "vouches" for the quality of the employee towards the employer.

which in turn improves economic efficiency.

2 Theory

This section presents a game-theoretic model of informal borrowing in social networks and shows that the value of an asset a borrower can obtain from a lender is bounded above by the *maximum network flow* (or *trust flow*) between borrower and lender. In Sections 3 and 4, where we consider applications and evidence, we only make use of this reduced form characterization of trust.

2.1 Model setup

In our model, a borrower needs an asset of a lender to produce social surplus. The asset might represent a factor of production, such as a farming tool, a vehicle or an animal in an agricultural economy; it could also be an apartment, a household durable good or simply a cash payment. In this economy, there is no formal contract enforcement that would prevent the borrower from stealing the asset after using it. As a result, the lender will be reluctant to lend unless some informal arrangement can be used to guarantee the return of the asset. In our model, the social network allows agents to provide informal contract enforcement: connections in the network have consumption value, which can be used as collateral to secure lending.

Formally, a social network $G = (W, E)$ consists of a set W of agents (vertices) and a set E of edges, where an edge is an unordered pair of distinct vertices. Each edge in the network represents a friendship or business relationship between the two parties involved. We formalize the strength of relationships using an exogenously given capacity $c(u, v)$.

Definition 1 *A capacity is a function* $c : W \times W \to \mathbb{R}$ *such that* $c(u, v) > 0$ *if* $(u, v) \in E$ *and* $c(u, v) = 0$ *otherwise.*

The capacity measures the utility benefits agents derive from their relationships. For ease of presentation, we assume that the strength of relationships is symmetric, so that $c(u, v) = c(v, u)$ for all u and v.

Our model consists of five stages, as depicted in Figure 2. We begin by describing the model, and then discuss the economic content of our modeling assumptions.

Stage 1: Realization of needs. Two agents s and t are randomly selected from the social network. Agent t, the lender, has an asset which agent s, the borrower, desires. The lender values the asset at V, and it is assumed that V is drawn from some prior distribution F over $[0, \infty)$. The identity of the borrower and the lender as well as the value of V are publicly observed by all players.

Stage 2: Borrowing arrangement. At this stage the borrower publicly proposes a *transfer arrangement* to all agents in the social network. The role of the transfer arrangement is to punish the borrower and compensate the lender in the event of default; this will ensure that even in the absence of courts the borrower returns the asset. A transfer arrangement consists of a set of transfer payments $h(u, v)$ for all u and v agents involved in the arrangement. Here $h(u, v)$ is the amount u promises to pay v if the borrower fails to return the asset to the lender. Once the borrower has announced the arrangement, all agents involved have the opportunity to accept or decline. If all involved agents accept, then the asset is borrowed and the borrower earns an income $\omega(V)$, where $\omega(.)$ is a non-decreasing function with $\omega(0) = 0$. If some agents decline, then the asset is not lent, and the game moves on directly to stage 5.

For large social networks it can be unrealistic to expect the borrower to include socially very distant agents in the borrowing arrangement. To formalize this, let $d(u, v)$ denote the length of the shortest path between u and v in the network, and define the distance between an edge (u, v) and a vertex s to be $[d(u, s) + d(v, s)]/2$.[6] We assume that the borrower s can only propose transfers $h(u, v)$ for those (u, v) links that are at most distance K away from him in the network. Here K is an exogenous parameter which we call the *circle of trust*. We allow K to be infinite, in which case the borrower can propose to all other agents in the network.[7]

Stage 3: Repayment. Once the borrower made use of the asset, he can either return it to the lender or steal it and sell it for a price of V.[8] If the borrower returns the asset then the game moves to the final stage 5.

Stage 4: Transfer payments. All agents observe whether the asset was returned in the previous stage. If the borrower did not return the asset, then the transfer arrangement is activated. Each agent has a binary choice: either he makes the promised payment $h(u, v)$ in full or he pays nothing. If some agent u fails to make a prescribed transfer $h(u, v)$ to v, then he loses his friendship with agent v (i.e., the (u, v) link "goes bad"). If an (u, v) link is lost, then the associated capacity is set to zero for the remainder of the game. We let $\widetilde{c}(u, v)$ denote the new link capacities after

[6] With this definition, the distance between s and (u, v) is either an integer or an integer divided by two.

[7] More generally, we could assume that s can propose transfers over all links in some subgraph G_s. Under this assumption, our main result would change in an intuitive way: the relevant maximum flow would have to be computed in G_s. We use the approach with K for ease of notation.

[8] The model can be extended to the case where the liquidation value of the asset is $\phi \cdot V$ with $\phi \leq 1$. We analyze this more general model in Appendix B.

these changes.

Stage 5: Friendship utility. At this stage, agents derive utility from their remaining friends. The total utility enjoyed by an agent u from his remaining friends is simply the sum of the values of all remaining relationships, i.e., $\sum_v \widetilde{c}(u,v)$.

2.2 Discussion of modeling assumptions

We now turn to discuss some of the assumptions underlying our model.

Cash bonds and borrowing constraints. The model is centered around an agency problem: in the absence of courts, the lender worries that the borrower might steal his asset. One way to solve this agency problem is to have the borrower s post a cash bond to the lender. The lender t returns the cash bond only if s returns the asset; otherwise the bond is kept as compensation. In the model, we abstract away from cash bonds and pre-payments by assuming that the borrower is initially completely cash-constrained. However, we do assume that in later stages the borrower and other agents are able to make certain payments. This can be justified if agents work or make investments in the initial stage, and they earn wages or return on their investment in later stages. Alternatively, payments in the game can represent in-kind transfers, e.g., helping out with the harvest, where posting a bond may be inefficient or infeasible.

Circle of trust. In the model, we parameterized the set of agents who can be involved in an informal borrowing arrangement with K, which we call the "circle of trust." Our motivation for this approach comes from Granovetter (1974), who, in his influential book on labor market networks, argued that job referrals typically involve less than three intermediaries. His findings suggest that the empirically relevant range for our circle of trust parameter is $0 < K \leq 2.5$.

Transfer arrangement as social norms. We think of the transfer arrangement in our model as a means of formalizing accepted norms of behavior. While in practice agents may not explicitly agree on transfer payments, often there is an understanding about their responsibilities in the case of default. For example, in Wechsberg's (1966) analysis of the prominent shipowner discussed in Section 1, there was presumably an implicit understanding between various agents about the course of action to take if the shipowner were to default.

Social sanctions. We model social sanctions by assuming that when an agent fails to make a promised transfer, the associated friendship link automatically goes bad. Loss of a friendship in this setup is not the result of a strategic decision; it is simply an assumption capturing the idea that friendly feelings often cease to exist if a promise is broken. We use this assumption to simplify the exposition of the model. However, it is possible to provide precise microfoundations for such behavior: failure to make a transfer might signal that an agent no longer values a particular friendship, in which case the former friends might find it optimal not to interact with each other in the future. Appendix B develops this idea formally, providing explicit microfoundations for loss of friendship as a social sanction.

2.3 Equilibrium Analysis

For what values of V can borrowing be implemented in a subgame perfect equilibrium? We begin answering this question by studying equilibria where all promises are kept, i.e., where every transfer $h(u,v)$ is expected to be paid if the borrower fails to return the asset. We later show that focusing on these equilibria is without loss of generality. In any equilibrium where promises are kept, transfers have to satisfy the capacity constraints

$$h(u,v) \leq c(u,v). \tag{1}$$

To see why, suppose that the borrower fails to return the asset, so that the transfer arrangement is activated, and consider some (u,v) link. Agent u now has to decide whether to make the payment $h(u,v)$ to v. The cost of making the payment is just its dollar value, i.e., $h(u,v)$; the cost of not making the payment is $c(u,v)$, because it results in losing a friendship of this value with v. Since we are focusing on equilibria where promises are kept, u must prefer the friendship over the monetary value of the transfer, i.e., (1) must hold.

We now turn to explore how the above capacity constraints lead to a representation of the borrowing limit as the maximum network flow in two simple networks.

Two-agent network. Let the social network consist of just two agents, the borrower s and the lender t, and consider a pure strategy equilibrium implementing borrowing where promises are kept. We first show that in any such equilibrium, $V \leq h(s,t)$ must hold. To see why, assume that the borrower s defaults on the equilibrium path. Then the lender receives the transfer payment $h(s,t)$ instead of the asset; but he must break even to lend, which yields $V \leq h(s,t)$. Now suppose instead that the borrower returns the asset on the equilibrium path. In this case, the borrower must weakly prefer not to default, which again requires $V \leq h(s,t)$.

Now combine $V \leq h(s,t)$ with the capacity constraint (1) to obtain

$$V \leq c(s,t). \tag{2}$$

This inequality characterizes the borrowing limit with the maximum network flow in the simple two-agent setup: borrowing can be implemented only if the asset value does not exceed the strength of the friendship $c(s,t)$, which also equals the maximum flow in this network. Intuitively, the collateral value of friendship can be used to elicit payment and solve the agency problem of lending. Conversely, when (2) is satisfied, it is easy to construct an equilibrium that implements borrowing: just set $h(s,t) = V$.[9]

Four-agent network. To gain intuition about the borrowing limit in more general networks, we next consider the network depicted in Figure 3, which consists of four players: the borrower s, the lender t, an intermediate agent u connecting s and t, and agent v who is only connected to the borrower s. We will refer to v as the "cousin" of s. A natural transfer arrangement that implements borrowing in this network is where agent u acts as an intermediary who elicits and transits payments from s to t in the case of no compliance, and gets zero net profits. In this arrangement, agent v plays no role. To formalize this arrangement, simply set $h(s,u) = h(u,t) = V$; that is, in the event of default, s is expected to pay V to u, who then transfers it on to t. For this arrangement to be feasible, it must satisfy the capacity constraint (1) for both links involved: $V \leq c(s,u)$ must hold so that s delivers the transfer to u, and $V \leq c(u,t)$ is needed to ensure that u passes on the transfer to t. Combining these, the candidate arrangement constitutes an equilibrium if and only if

$$V \leq \min\left[c(s,u), c(u,t)\right]. \tag{3}$$

The content of this result is that the "weakest link" on the path connecting s to t determines the maximum borrowing limit. Inequality (3) thus establishes that the maximum flow determines the borrowing limit for this class of transfer arrangements.

However, networks with more than two agents generally admit other transfer arrangements, which can implement borrowing even if (3) fails. To take one example, assume that the borrower s has a strong link to his cousin v, with a capacity value of $V + 1$. The borrower might then propose

[9] In this equilibrium, all surplus accumulates to the borrower because of our assumption that he proposes the transfer arrangement. In a setup where bargaining power is more evenly distributed, we expect that the surplus would be shared by the agents involved in the transfer arrangements, in a manner similar to Goyal and Vega-Redondo (2004).

an informal arrangement in which he promises to pay his cousin a transfer of $h(s, v) = V + 1$ in case he fails to return the asset. This arrangement provides the right incentives to the borrower, and is an equilibrium even if (3) fails. To understand its logic, note that in this arrangement, the borrower essentially makes the following proposal to the lender: "Lend me your asset; if I don't return it to you, my cousin will be angry with me." As this interpretation makes it clear, this borrowing arrangement may not be robust to joint deviations where both the borrower and his cousin depart from equilibrium. More concretely, the borrower could circumvent the arrangement by entering a *side-deal* with his cousin, in which he steals the asset and shares the proceeds with the cousin (who in equilibrium would otherwise receive nothing). Doe to the possibility of such side-deals, we do not find this equilibrium plausible.

A similar potential equilibrium is one where the intermediate agent u provides incentives to the borrower but promises a zero transfer to the lender. In this case, the lender effectively "outsources" monitoring to the intermediate, trusting that the borrower will always return the asset rather than pay a high transfer to u. This arrangement is again open to side-deals: here s and u can choose to steal the asset jointly and split the proceeds, leaving the lender with nothing. As in the equilibrium with the cousin, the possibility of a side-deal arises because nobody "monitors the monitorer": the lender is not fully in control of incentives. When enforcement is outsourced to either the cousin or the intermediary, these agents can "team up" with the borrower and steal the asset.

These examples suggest that when the borrower and other agents can agree to side-deals, it may not be in the interest of the lender to provide the asset. This motivates our focus on equilibria that are immune to such side-deals. The requirement of side-deal proofness will ensure that the lender always maintains full control of incentives if he agrees to lend the asset.

2.4 Side-deal proof equilibrium

Formally, a side-deal consists of an alternative transfer arrangement $\widetilde{h}(u, v)$ offered by s to a subset of agents $S \subset W$. If a side-deal is accepted, agents in S make transfer payments according to $\widetilde{h}$, while agents outside S continue to make payments described by h. In order for the side-deal to be credible to all participating agents, it must be accompanied by a proposed path of play that these agents find optimal to follow. This motivates the following definition.

Definition 2 *Consider a pure strategy profile σ. A side-deal with respect to σ is a set of agents S, a transfer arrangement $\widetilde{h}(u, v)$ for all $u, v \in S$, and a set of continuation strategies $\{\widetilde{\sigma}_u | u \in S\}$*

proposed by s to agents in S at the end of stage 2, such that

(i) $U_u(\widetilde{\sigma}_u, \widetilde{\sigma}_{S-u}, \sigma_{-S}) \geq U_u(\sigma'_u, \widetilde{\sigma}_{S-u}, \sigma_{-S})$ *for all* σ'_u *and all* $u \in S$,

(ii) $U_u(\widetilde{\sigma}_S, \sigma_{-S}) \geq U_u(\sigma_S, \sigma_{-S})$ *for all* $u \in S$,

(iii) $U_s(\widetilde{\sigma}_S, \sigma_{-S}) > U_s(\sigma_S, \sigma_{-S})$.

Condition (i) says that all agents u involved in the side-deal are best-responding on the new path of play. This condition implies that the proposed path of play is an equilibrium for all agents in S conditional on all others playing their original strategies σ_{-S}. Condition (ii) says that if any agent $u \in S$ denies participating in the side-deal, then play reverts to the original path of play given by σ. Finally, (iii) ensures that the borrower s strictly benefits from the side-deal.

Definition 3 *A pure strategy profile* σ *is a side-deal proof equilibrium if it is a subgame perfect equilibrium that admits no side deals.*

Side-deal proofness allows us to exclude any equilibrium for the network in Figure 3 where the weakest-link inequality (3) fails. Our main theorem establishes this claim for general networks.[10]

2.5 Main theorem

We begin by formally defining the concept of network flows intuitively discussed above.

Definition 4 *An* $s \to t$ *flow with respect to capacity c is a function* $f : G \times G \to \mathbb{R}$ *which satisfies*

(i) Skew symmetry: $f(u, v) = -f(v, u)$.

(ii) Capacity constraints: $f(u, v) \leq c(u, v)$.

(iii) Flow conservation: $\sum_w f(u, w) = 0$ *unless* $u = s$ *or* $u = t$.

The value of a flow is the amount that leaves the borrower s, given by $|f| = \sum_w f(s, w)$. A K-flow is a flow such that $f(u, v) > 0$ implies that the distance between the borrower s and the edge (u, v) is at most K, the circle of trust. Let $T_K^{st}(c)$ denote the maximum value among all $s \to t$ K-flows.[11] Our main theorem extends inequalities (2) and (3) to general networks.

[10] Our definition of side-deal proof equilibrium does not require side-deals to be robust to further side-deals. This distinction is immaterial in our model: it is easy to show that requiring side-deals to be robust to further side-deals does not change any of the results in this paper.

[11] The maximum exists because flow value is a continuous function and the set of K-flows is a compact subset of a finite-dimensional Euclidean space.

Theorem 1 *There exists a side-deal proof equilibrium that implements borrowing between s and t if and only if the asset value V satisfies*

$$V \leq T_K^{st}(c). \tag{4}$$

This result states that the endogenous borrowing limit equals the value of the maximum K-flow between borrower s and lender t. When V satisfies this inequality, a side-deal proof equilibrium is easy to construct: by assumption, there exists an $s \to t$ flow with value V, and this flow can be used as a transfer arrangement. Flow conservation implies that all intermediate agents break even, confining their role to simply extracting and transmitting the payment V from s to t in case s fails to return the asset. Thus the lender effectively controls the provision of incentives; because of this, the equilibrium is easily seen to be side-deal proof.

To show that no side-deal proof equilibrium can implement a higher level of borrowing, we build on the maximum flow-minimum cut theorem (Ford and Fulkerson 1956), which states that the maximum network flow between borrower s and lender t equals the value of the minimum cut. A cut is a disjoint partition of the vertices into two sets $G = S \cup T$ such that $s \in S$ and $t \in T$, and the value of the cut is defined as the sum of $c(u, v)$ for all links such that $u \in S$ and $v \in T$.

For any borrowing arrangement violating (4), we can construct a side-deal using the set of agents S in a minimum cut. To see the logic, note that the amount transferred between S and T cannot exceed the value of the cut. Given that (4) fails, this implies that it is not possible for the full value V to flow from S to T, and hence agents in S as a group pay less than V in the event of default. But this means that agents in S as a group do not have the right incentives to return the asset; as a result, they can deviate as a group, steal the asset and split the proceeds among themselves.

2.6 Extensions: multiple loans and transfer constraints

In practice, the assumption of a single borrower and a single lender is often restrictive. Similarly, in environments with severe credit constraints, agents might have limits on the total amount of transfers they can make. In this section, we address these problems by considering two extensions of the model: allowing for multiple borrowers and lenders, and introducing constraints on agents' total transfers. We show that the concept of network flows can be used to characterize borrowing in both extensions: Borrowing limits can be computed as the maximum flow in certain auxiliary networks. These results show that network flows can be useful for the study of informal contract enforcement in more realistic settings as well.

Multiple borrowers and lenders. Assume that the asset agents wish to borrow from each other is money. Let s_1, s_2,..., s_k be the set of borrowers, and suppose that agent s_i would like to borrow V_i dollars. The set of lenders who can supply loans is t_1, t_2..., t_m, where agent t_j can lend at most W_j dollars. Each borrower can borrow from multiple lenders if necessary, and borrowers and lenders do not care about the identity of their counterparts as long as the get the desired loan and are expected to be repaid.[12]

When can all borrowers simultaneously borrow their desired amounts? To answer this question, we define an auxiliary directed network G', which is constructed in the following way. We start with the network G, and replace each (u, v) link with a pair of two directed links: an $u \to v$ link with capacity $c(u, v)$; and a $v \to u$ link that also has capacity $c(u, v)$. Next we add two new vertices, denoted by s_0 and t_0; and for each borrower s_i, $i \geq 1$, add a directed link $s_0 \to s_i$ with capacity $c(s_0, s_i) = V_i$, and for each lender t_j, $j \geq 1$, add a directed link $t_j \to t_0$, with capacity $c(t_j, t_0) = W_j$.

As we formally demonstrate in the Appendix, full borrowing can be achieved if and only if the maximum network flow between s_0 and t_0 in the directed network G' is at least as high as the total desired loan amount, $V_1 + ... + V_k$. To understand the intuition, note that the neighbors of the new vertex s_0 are the set of borrowers, and the neighbors the new vertex t_0 are the set of lenders. As a result, any $s_0 \to t_0$ flow can be interpreted as originating in the set of borrowers and transferring resources to the set of lenders. A maximal $s_0 \to t_0$ flow can then be used to construct an equilibrium implementing borrowing: The amount carried from borrower s_i to lender t_j in the flow specifies how much s_i will borrow from t_j; and the flow values over links define the transfer payments analogously to the single borrower and single lender setup.

If the maximum $s_0 \to t_0$ flow has value $V_1 + ... + V_k$, then the above construction clearly satisfies the aggregate demand for loans. But we still need to verify that each borrower gets his exact desired amount, and also that the resource constraints of lenders are not exceeded. Both of these properties follow from the construction of G'. To see why, note that the capacities of the links originating in s_0 are defined by the loan demands of the borrowers: link $s_0 \to s_i$ has capacity V_i. This means that the links of s_0 have a total capacity of $V_1 + ... + V_k$, and therefore any $s_0 \to t_0$ flow that carries this amount must use each of these links at full capacity. But then our candidate

[12] In this analysis, we abstract away from the restrictions imposed by the "circle of trust" by assuming that $K = \infty$.

equilibrium allocates exactly V_i dollars to borrower s_i. Similarly, the capacity constraint W_j over the $t_j \rightarrow t_0$ link for each j ensures that no lender provides loans exceeding his available funds, and hence the candidate equilibrium satisfies all resource constraints.

Transfer constraints. We now return to the single borrower and single lender setup, but introduce constraints agents' total transfers. Suppose that each agent u can transfer at most a total of k_u to others in the network, where the "transfer constraints" k_u are exogenous. A natural interpretation is that k_u represent time constraints. If transfers are in-kind services such as helping out, then they require time, and available time limits how much agents can transfer. Moreover, the utility gain from an in-kind transfer cannot be easily passed on to another agent, and hence incoming transfers need not relax the time constraint.[13]

How much borrowing can be implemented in this environment? As above, constructing an auxiliary directed network G'' will help answer this question. We begin by replacing each node u in G with a pair of two nodes, u_1 and u_2. Next, we replace each (u, v) link with two new directed links: an $u_2 \rightarrow v_1$ link and a $v_2 \rightarrow u_1$ link, both with capacity equal to $c(u, v)$. Finally, for each agent u we create a new $u_1 \rightarrow u_2$ link with capacity equal to the transfer constraint $c(u_1, u_2) = k_u$. The idea is to duplicate all agents u, then point all incoming links of u to u_1, have all outgoing links of u originate in u_2, and let the capacity of the $u_1 \rightarrow u_2$ link be determined by the transfer constraint k_u.

In the Appendix we show that in any side-deal proof equilibrium where promises are kept, the borrowing limit in the presence of transfer constraints equals the value of the maximum $s_1 \rightarrow t_1$ flow in G''. To understand the intuition, consider a maximal flow. As in the basic model, the amounts assigned to links between agents by this flow can be interpreted as the transfer payments in a candidate transfer arrangement. It remains to verify that in this arrangement, no agent u exceeds his total transfer constraint k_u. But this follows by construction of G''. The total amount of transfers promised by u must be equal to the flow leaving u_2 in G''; but by flow conservation, this must be equal to the value carried over the $u_1 \rightarrow u_2$ link, which is bounded by the link capacity of k_u in G''.

[13] The k_u can also be thought of as a reduced from representation of cash constraints. While incoming transfers might relax cash constraints, an extended model where agents have limited cash and transfers fail with small probability can lead to similar reduced form constraints.

3 Applications

We begin with some preliminaries. Theorem 1 established that in our baseline model, the borrowing limit equals the maximum network flow $T_K^{st}(c)$ between borrower s and lender t. We interpret the borrowing limit as a measure of trust, and refer to the function $T_K^{st}(c)$ as the trust map. To see how the trust measure $T_K^{st}(c)$ is related to payoffs, let $\Pi_K^{st}(c)$ denote the expected payoff of s from borrowing, conditional on the lender being agent t:

$$\Pi_K^{st}(c) = \Pi\left(T_K^{st}(c)\right) \text{ where } \Pi(z) = \int_0^z \omega(v) \; \mathrm{d}F(v), \tag{5}$$

because the expected payoff of s is just the expectation of $\omega(V)$ over all values of V that do not exceed the borrowing limit (4). Equation (5) shows that the expected payoff from borrowing is a monotone increasing function of trust.[14]

We now introduce measures of trust at the level of the individual and the community. We define the level of trust enjoyed by a borrower s to be

$$T_K^s(c) = \sum_{t \in W} T_K^{st}(c).$$

This measure is proportional to the average trust of s, $T_K^s(c)/(N-1)$, which has a natural interpretation as the expected level of trust conditional on s being the borrower who needs the asset of a randomly selected lender. We use the sum rather than the average for expositional reasons. Note that $T_K^s(c)$ can be large both if s has low pairwise trust with many people, and also if s has high pairwise trust with fewer people. $T_K^s(c)$ summarizes the distribution of pairwise trust levels between s and other agents through its mean. In Section 3.3, we explore how higher order properties of this distribution affect welfare.

Analogously, define the community-wide trust embedded in the social network as

$$T_K(c) = \sum_{s \in W} T_K^s(c).$$

Community-wide trust is proportional to the average level of trust between a randomly selected borrower-lender pair. Using similar notation, we denote the expected payoff from borrowing of an individual s by $\Pi_K^s(c)$, and the sum of expected payoffs from loans of all agents by $\Pi_K(c)$.

[14] Besides the payoff from borrowing $\Pi_K^{st}(c)$, the borrower s also derives utility from his friends. As a result, the total utility of the borrower U_s always satisfies $U_s \geq \Pi_K^{st}(c)$.

The theoretically correct measure of welfare generated by borrowing is $\Pi_K(c)$, which summarizes the expected utility of agents. However, the trust measure $T_K(c)$ has the advantage that it can be computed using only network data, without knowledge of the distribution function F and the payoff function ω. As equation (5) shows, in general there is a distinction between these measures even at the level of borrower-lender pairs. A useful special case when the two measures are equivalent is when the function $\Pi(z)$, which maps trust into payoffs, is linear, i.e., $\Pi(z) = \pi \cdot z$ for some $\pi > 0$. In this *linear benchmark case*, trust and payoffs are proportional both at the level of the individual and at the level of society.[15]

Two additional cases of interest are when the linearity assumption is relaxed, but some restrictions are still placed on the on the curvature of $\Pi(z)$. We say that technology favors *high asset values if* $\Pi(z)$ is convex. In this case, more weight is given to assets with relatively high value, in the sense that either (1) they are more probable, or (2) they generate higher surplus $\omega(z)$. Conversely, technology favors *low asset values* if $\Pi(z)$ is concave, in which case low asset values have relatively higher probability or associated surplus. With this classification, the linear benchmark case can be viewed as technology being neutral with respect to asset values: low asset values and high asset values have equal weight. We will make use of this classification in the analysis below.

3.1 Comparative statics

Here we explore the comparative statics of trust and payoffs with respect to changes in the social network. We vary the network structure by changing the capacity c which measures the strength of links between agents. Such changes can also be interpreted as changing the underlying network, because a link with capacity zero is equivalent to having no link at all. To formally analyze comparative statics, let the space of all symmetric capacities on the set of agents W be denoted by $\mathcal{C} \subset \mathbb{R}^{W \times W}$. Then $T_K^{st}(c)$ can be interpreted as a map $T_K^{st} : \mathcal{C} \to \mathbb{R}$, which computes trust between s and t for different networks. The following result shows that trust is a well-behaved function of the social network.

Proposition 1 *[Properties of trust map] The map $T_K^{st}(c)$ is nondecreasing and concave in c and nondecreasing in K.*

$T_K^{st}(c)$ is nondecreasing because a feasible flow cannot be rendered infeasible by increasing the capacities of some links. Concavity follows since the convex combination of feasible flows for

[15] The linear benchmark can be obtained, for example, by assuming that the benefit function $\omega(V) = \omega$ is a constant, and that the distribution of asset values F is uniform on a $[0, a]$ interval such that $a > \max_{s,t} T_K^{st}(c)$.

different capacities is a feasible flow under the convex combination of the capacities.

This proposition leads directly to our main comparative statics results. Before stating the results, we introduce some terminology. We say that the network associated with capacity c_1 is *more connected* than the network associated with capacity c_2 if no link has lower capacity under c_2 than under c_1: that is, $c_1(u,v) \geq c_2(u,v)$ for all $u, v \in W$. Let $\widetilde{C}_1$ and $\widetilde{C}_2$ be two capacity-valued random variables, that is, two random networks. We say that $\widetilde{C}_2$ is *more heterogenous* than $\widetilde{C}_1$ if the probability distribution of $\widetilde{C}_1$ second-order stochastically dominates that of $\widetilde{C}_2$. To see the intuition, note that second order stochastic dominance captures the idea of more uncertainty for random networks the same way as it does for random scalars. In words, $\widetilde{C}_2$ can be viewed as a probability distribution with "fatter tails" that assigns greater weight to more heterogenous and extreme networks. For example, if the capacities over links are independent, the random network $\widetilde{C}_2$ can be obtained from $\widetilde{C}_1$ by adding mean-preserving spreads to the capacity of each link.

We are now ready to state our main comparative statics result.

Corollary 1 *[Trust comparative statics] For every borrower s and lender t and any K,*

(i) [Monotonicity] If a social network with capacity c_1 is more connected than the network with capacity c_2, then both trust and payoffs are higher: $T_K^{st}(c_1) \geq T_K^{st}(c_1)$ and $\Pi_K^{st}(c_1) \geq \Pi_K^{st}(c_1)$.

(ii) [Heterogeneity] If a random network $\widetilde{C}_1$ is less heterogenous than a random network $\widetilde{C}_2$, then trust is higher: $E\Big[T_K^{st}(\widetilde{C}_1)\Big] \geq E\Big[T_K^{st}(\widetilde{C}_2)\Big]$. Moreover, if technology favors low asset values (i.e., $\Pi(z)$ is concave) then expected payoffs are higher as well: $E\Big[\Pi_K^{st}(\widetilde{C}_1)\Big] \geq E\Big[\Pi_K^{st}(\widetilde{C}_2)\Big]$.

This result, which follows immediately from Proposition 1, summarizes the basic comparative statics properties of trust.[16] Part (i) shows that networks with more and stronger links generate higher levels of trust, and consequently higher payoffs to all agents. This result supports Putnam's (1995) general argument about the role of social connections in increasing trust. The precise content of part (ii) is that if a network is drawn from a distribution that assigns greater weight to more heterogenous and extreme networks, expected trust falls. This result can be interpreted as stating that *on average*, trust levels are lower in more heterogenous societies. The intuition is straightforward: in our model, trust is determined by the strength of the weakest link along certain paths. When heterogeneity increases, the weakest link becomes weaker on average, leading to lower trust.

[16] In a different setup, Galeotti, Goyal, Vega-Redondo, and Yariv (2006) explore comparative statics of equilibria in network games where the degree distribution across networks is related by first order or second order stochastic dominance.

To connect the result about heterogeneity and trust to stylized facts, it is useful to consider a simple model of ethnic or racial heterogeneity. Fix a social network G, and assume that all links in the network have the same capacity 1. Now color each agent in the network independently and with equal probabilities either black or white; and modify the capacities such that the strength of a link between a black and a white agent is now $1 - x$, while the strength of a link connecting agents of the same color is $1 + x$. For each x between 0 and 1, this construction generates a random network where the expected capacity of all links is 1. Here x can be interpreted as a measure of heterogeneity: it is easy to see that a higher x results in a more heterogenous network according to our definition. As a result, part (ii) of the Corollary can be applied to show that average trust between two agents is monotonically decreasing in heterogeneity x. Note that trust here averages over both pairs of agents with different color as well as pairs with the same color; any potential increase in trust within a group is more than compensated by the loss of trust across groups.

These theoretical results about social heterogeneity and trust are consistent with existing evidence. Alesina and La Ferrara (2002) show that living in a U.S. locality that is racially mixed or displays a high degree of income disparity is correlated with low average trust.[17] Putnam (2007) shows in data from the Social Capital Benchmark Survey panel that in ethnically diverse US communities there is not only lower trust between people from different backgrounds but even lower trust between people of the same background. This finding is compatible with our model if indirect links are an important source of trust in a community. Leigh (2006) finds similar results in Australian survey data. Finally, Knack and Keefer (1997) show that trust is weaker in nations with more income inequality, which is consistent with our results if income inequality results in social stratification of the kind represented by the random network example of the previous paragraph.

Due to the distinction between trust and payoffs, lower trust generated by heterogeneity does not always translate into lower payoffs; as the Corollary shows, for this result we need technology to favor low asset values. Intuitively, if high asset values were relatively likely, greater heterogeneity could increase expected payoffs by giving rise to strong paths between borrower and lender with small probability. An extreme example is when asset values are always higher than some threshold. Then, a homogenous network can lead to no borrowing if the network flow between any pair of agents is below the threshold; but in a heterogenous network some connections might be strong enough to make borrowing feasible.

[17] In related work, Alesina and La Ferrara (2000) show that more heterogenous societies exhibit lower participation rates in social activities.

3.2 Simple trust measures

In this section we explicitly compute the trust flow $T_K^{st}(c)$ for small values of the circle of trust K, and express the resulting simple trust measures in terms of commonly used network statistics. These results are useful because (1) they show that our simple measures can be computed with limited network data which is often available in existing datasets, and (2) they provide microfoundations for standard network statistics in terms of trust and social collateral.

We begin by introducing some standard network statistics. For any agent s in the network, denote the set of his direct neighbors by N_s and call $|N_s|$ the *degree* of s. Then the total number of edges $|E|$ can be expressed as $|E| = (1/2)\sum_s |N_s|$. The next three statistics measure the extent to which the neighborhoods of different agents overlap. The *clustering coefficient* κ_s is simply the number of links between s's friends divided by the maximum possible number of such links:

$$\kappa_s = \frac{\#\text{ of links between agents in } N_s}{|N_s|\left(|N_s|-1\right)/2}.$$

This is a common measure of local network density used in diverse fields including sociology, physics and economics (e.g. Watts and Strogatz 1998, Albert and Barabasi 2002, Jackson 2005).[18] For any two agents s and t, let n_{st} denote the *number of common friends* that they have, a statistics used by Glaeser, Laibson, Scheinkman, and Soutter (2000) and De Weerdt (2002) to proxy for the ability to enforce cooperative behavior. Finally, let Δ denote the *number of triangles* in the social network, a quantity related to a measure of clustering used in sociology called the "fraction of transitive triples" (Wasserman and Faust 1994).[19]

Before proceeding, we need to introduce one additional variable, that measures the degree to which an agent's neighborhood is interconnected through third parties. Let I_{uv}^s be an indicator variable that equals one if u and v have a common friend who is not a friend of s. This variable can be interpreted as measuring the presence of 4-cycles locally around agent s in the network.

The next result expresses trust flows using the network statistics introduced above. In this analysis, we only focus on networks where all links are equally strong, and normalize the capacity of each link to one; this means that $c(u,v) = 1$ for all $(u,v) \in E$.

Proposition 2 *[Simple trust measures] The following are true*

(i) If $K = 0.5$ then $T_K^{st}(c) = 1_{\{t\in N_s\}}$, $T_K^s(c) = |N_s|$ and $T_K(c) = 2|E|$.

[18] When $|N_s| \leq 1$, we let $\kappa_s = 0$.

[19] A triangle is a set of three vertices u_1, u_2 and u_3 such that there is a link between any two of them.

(ii) If $K = 1$ *then* $T_K^{st}(c) = [1 + n_{st}] \cdot 1_{\{t \in N_s\}}$, $T_K^s(c) = |N_s|\,(1 + (|N_s| - 1)\,\kappa_s)$ *and* $T_K(c) = 2\,|E| + 6 \cdot \Delta$.

(iii) If $K = 1.5$ *then* $T_K^{st}(c) = n_{st} + 1_{\{t \in N_s\}} \cdot \left[1 + \sum_{u \in N_s} I_{ut}^s\right]$ *and* $T_K^s(c) = \sum_{t \in N_s} |N_t| + \sum_{u,v \in N_s} I_{uv}^s$.

Part (i) states that when $K = 0.5$, so that the circle of trust only involves direct friends, the trust enjoyed by s is determined by his degree $|N_s|$, and community-wide trust is proportional to the number of links $|E|$ in the network. For such small K, indirect paths do not increase trust, and the average trust of an agent is determined by his "access" measured by the number of his friends.

As we raise the circle of trust K to 1, indirect paths begin to matter. Trust between two connected agents s and t now depends on the number of their common friends n_{st}: each common friend can vouch for the borrower and provide additional leverage, increasing the borrowing limit. Trust enjoyed by an individual $T_K^s(c)$ depends both on the size of his neighborhood and its density, measured by the clustering coefficient κ_s. A small but densely connected neighborhood can be as beneficial as a large number of friends who are not connected to each other. Intuitively, while the likelihood of finding a lender in small and dense neighborhood is low, the borrowing limit for any potential lender is high. At the population level, trust depends both on the total number of links $|E|$ which measure access to resources, and on the total number of triangles Δ which measure the density of connections.

When $K = 1.5$, all paths of length at most 2 generate one unit of trust. As a result, when s and t are at distance two from each other, pairwise trust equals n_{st}, the number of their common friends. When s and t are also friends, pairwise trust increases for two reasons. First, their direct connection adds one unit of trust; second, there may be a 4-cycle involving both s and t that further increases trust. The latter effect is captured by the I_{ut}^s terms in the expression for pairwise trust. The total trust enjoyed by s is a function of both the sizes of neighborhoods of his friends and the degree to which these are interconnected through 4-cycles.

The Proposition show that for K small, our simple trust measures only depend on the immediate network neighborhood around agent s. This result can be significant for applied research. In practice, complete network data is rarely available, but many existing datasets contain limited information about the immediate network around respondents. For example, the 1985 module of the General Social Survey (GSS) collects data about subjects' close friends and the relations between those friends. Similarly, sociologists using the "survey-network research design" often collect network data about direct connections and the relations between them (Granovetter 1976, Burt 2006). With such data, our simple trust measures can be computed for both $K = 0.5$ and $K = 1$. This opens the possibility of exploring the relationship between our trust measures and outcomes related to trusting behavior in practice.

The Proposition also provides foundations for common network statistics in terms of social collateral. Our foundations can be used both to interpret and to qualify these network statistics. For example, part (ii) of the Proposition shows that the clustering coefficient κ_s is an imperfect measure of social collateral. According to our model, one should augment κ_s with the size of the agent's neighborhood, because a larger neighborhood, even if it has lower clustering, can increase access and lead to greater social collateral.

Of course, our results have limitations as well. For example, assuming that all friendship capacities are equal might be unreasonable in some applied settings. In such environments, we need information about the strength of relationships (perhaps proxied by the amount of time agents spend with each other) to compute flow measures of trust.

3.3 Closure vs. structural holes

Proposition 2 showed how to compute trust as a function of the network structure. However, as we discussed in Section 3.2, expected payoffs can vary even if the level of trust is held fixed. In this section, we explore which network structures yield higher payoff for a given level of trust.

The sociology literature on networks has two broad views about which types of social networks are better. One view, dating back to Coleman (1988), stresses the importance of dense networks, referred to as network closure. A key element of Coleman's argument is that closure provides social capital because it facilitates sanctions that make it easier for individuals to trust each other. For example, in his discussion of the wholesale diamond market in New York City, Coleman explains that "If any member of this community defected through substituting other stones or stealing stones in his temporary possession, he would lose family, religious and community ties." His view can be illustrated with the two network neighborhoods depicted in Figure 4, which are identical to those used in Figure 1 of Coleman (1988). Intuitively, the neighborhood in Figure 4b has higher closure, because t_1 and t_2 are connected to each other. In the context of this figure, Coleman argues that higher closure is beneficial, because if the actions of s need to be constrained, in Figure 4b agents t_1 and t_2 can "combine to provide a collective sanction, or either can reward the other for sanctioning."

In contrast, Burt (1995) emphasizes the role of structural holes, that is, people who bridge otherwise disconnected networks. For example, agent s is a structural hole in Figure 4a, but not in Figure 4b. Burt's view suggests that loose networks and wide neighborhoods, i.e., low network closure like in Figure 4a, lead to higher performance. He argues that structural holes "broker the flow of information between people, and control the projects that bring together people from opposite sides of the hole" (Burt 2000). A key part of his argument is that loose networks provide easier access to information and other resources.

To evaluate the relative advantage of closure versus structural holes in our model, we first need a measure of closure. To construct such a measure, consider Figure 4, let all links have capacity 1, and let $K \geq 1.5$. Under these assumptions, in our model both neighborhoods in the Figure generate the same trust of 4 for agent s.[20] The network in Figure 4a achieves this by providing access to four agents, but only allows agent s to borrow "small assets" of value at most 1 from any of them. In contrast, the network in Figure 4b generates the same trust by providing access to only two agents, but allowing s to borrow "large assets" of value at most 2 from either of them. This analysis suggests that holding fixed the trust of s, higher closure can be interpreted as s having high pairwise trust with a small number of agents, while lower closure means having low pairwise trust with a large number of agents.

We can formalize these ideas the following way. Assume that the capacity function is integer valued, fix agent s, and consider the distribution of pairwise trust between s and all other agents in the network. Let $\tau_s(j)$ denote the number of agents with whom pairwise trust equals j for $j = 1, 2, ..., N-1$. For each j, the contribution of agents with pairwise trust j to the total trust of s is simply $j \cdot \tau_s(j)$, which we denote by $q_s(j)$. With this notation, the total trust of s can be decomposed as

$$T_K^s(c) = \sum_j q_s(j).$$

Holding fixed trust $T_K^s(c)$, changes in the network structure around s are reflected in changes in the $q_s(j)$ terms. For example, the network in Figure 4a with low closure has $q_s(1) = 4$ and $q_s(2) = 0$, while the network in Figure 4b with high closure has $q_s'(1) = 0$ and $q_s'(2) = 4$. Increasing closure thus corresponds to moving weight from $q_s(j)$ terms with low j to $q_s(j)$ terms with high j, i.e., increasing the relative contribution of agents with high pairwise trust to the total trust of s. Viewing the q_s function as a (scaled) probability density function, higher network closure can then be interpreted as moving weight to the right of the distribution, or equivalently, an increase in the

[20] In Figure 4a, pairwise trust $T_K^{st_i}(c) = 1$ for all four t_i, which adds up to $T_K^s(c) = 4$. In Figure 4b, pairwise trust is $T_K^{st_i}(c) = 2$ for t_1 and t_2 for a total of 4.

sense of first order stochastic dominance. This motivates the following definition.

Definition 5 *The neighborhood of s has a higher closure than the neighborhood of s' if*

(i) $T_K^s(c) = T_K^{s'}(c)$ so that both neighborhoods generate the same total trust; and

(ii) the probability distribution with density $q_s(j)/T_K^s(c)$ dominates the distribution with density $q_{s'}(j)/T_K^{s'}(c)$ in the sense of first order stochastic dominance.

This concept of network closure allows us to evaluate whether high closure or low closure is associated with higher expected payoffs in our model. The next result shows that the answer depends on the type of assets borrowed in the social network. Recall our definition that technology favors low value assets if $\Pi(z)$ is concave, which corresponds to assets of smaller value being more probable or generating higher surplus. Conversely, we say that technology favors high value assets if $\Pi(z)$ is convex, i.e., if high-value assets are either more likely or have greater surplus.[21]

Proposition 3 *If the technology favors high value assets, then a neighborhood with higher closure leads to a higher expected payoff to s. Conversely, if the technology favors low value assets, then a neighborhood with higher closure leads to a lower expected payoff to s.*

The basic intuition can be understood in terms of a trade-off between access and pairwise trust. When technology favors low asset values, the loose social network in Figure 4a is more profitable, because this network provides *greater access* to s: knowing more people directly or indirectly increases the likelihood that he obtains a low-value asset from some other agent. This logic is in line with Burt's basic argument that structural holes facilitate access to relatively cheap assets, such as information or advice.[22] The result also suggests that a more dispersed social network is more advantageous if it is mainly used to exchange small favors, which is consistent with Granovetter's (1973) argument about the strength of weak ties.

In contrast, when technology favors high asset values, a neighborhood like in Figure 4b leads to higher payoff. Here, access is reduced because the neighborhood is smaller, but this is more than compensated for by the fact that through his dense connections s will be able to borrow even high-valued assets, which are now relatively more attractive. This logic is in line with Coleman's general

[21] If F is differentiable, then $\Pi'(z) = f(z)\,\omega(z)$. If Π is convex than $f(z)\,\omega(z)$ must be increasing in z, meaning that either $f(z)$ or $\omega(z)$ must be larger for larger z; i.e., more valuable assets are either more likely or generate greater surplus.

[22] Exchange of information is not explicitly incorporated in our model. However, providing information e.g., about a job, is often an activity that has some small cost to the sender. Such information can be interpreted as a favor for which the sender might expect compensation. This logic is consistent with the basic intuition underlying our borrowing model.

argument for network closure, and particularly his example of the wholesale diamond market in New York City, where the exchange of highly valuable stones requires high trust between dealers.[23]

The results of the Proposition are related to earlier work on social capital in both sociology and in economics. In sociology, Putnam (2000) introduced the ideas of bonding and bridging social capital. In Putnam's view, bonding social capital is associated with dense social networks and is good for generating reciprocity between agents who know each other well. As an example, Putnam cites ethnic enclaves, who, among other things, provide "start-up financing, markets, and reliable labor for local entrepreneurs." In contrast, the networks underlying bridging social capital are "outward looking and encompass people across diverse social cleavages." Such bridging networks are good for "linkage to external assets and for information diffusion." The intuition behind the Proposition is similar to Putnam's argument: strong links and dense networks facilitate big favors and thus generate bonding social capital, while weak links are good for maximizing access which creates bridging social capital. Our model can thus be interpreted as providing formal foundations for these two aspects of social capital. In economics, Sobel (2002) made a similar point when he argued that "widely scattered weak links are better for obtaining information, while strong and dense links are better for collective action." We do not formalize the exchange of information, but the intuition behind the Proposition is similar to Sobel's general point.

4 Social collateral and trust: Evidence from dictator games

In this section, we present new evidence consistent with the quantitative implications of our model. Our data consists of one treatment from a larger web-based experiment with 569 Harvard undergraduates conducted by Markus Mobius and Tanya Rosenblat, analyzed in more detail in Leider, Mobius, Rosenblat, and Do (2006). We consider the behavior of subjects in two-player dictator game experiments, and relate the outcomes to network flow measures of trust.[24]

Background and estimation framework. In the specific dictator game we use, player 1, the allocator, can choose how to distribute a limited budget of tokens between himself and player 2, the recipient. Each token is worth three times as much to the recipient than to the allocator, which implies that the socially efficient action is to give all tokens to the recipient.

When played in isolation, the dictator game has a unique Nash equilibrium in which the allocator

[23] Vega-Redondo (2005) reports a similar finding in a model of repeated games played in networks: he shows that stability of cooperative behavior depends on a certain measure of network cohesiveness.

[24] Starting with Berg, Dickhaut, and McCabe (1995), there is a large experimental literature on trusting behavior in games, including Fershtman and Gneezy (2001) and Andreoni and Miller (2002).

keeps all tokens to himself. However, agents who trust each other could achieve a more efficient allocation: e.g., the allocator may expect compensation after the game either through monetary payments or other favors. If agents do expect post-play compensation, then the dictator game experiment closely corresponds to the borrowing model of this paper. The allocator plays the role of a lender, who, by giving some tokens to the recipient, effectively lends a monetary asset. Following the game, the allocator expects compensation, just like the lender does in our model. While the analogue with the model is incomplete in that there is no transfer arrangement signed by the subjects, in practice we expect that social norms of engagement govern subjects' behavior and substitute for explicit transfer arrangements.

With this interpretation, our model implies a negative relationship between the network flow measure of trust and the number of tokens kept by the allocator. This motivates the estimating equation

$$\text{selfish behavior}_{st} = \alpha + \beta \cdot \text{trust flow}_{st} + \text{controls} + \varepsilon_{st} \tag{6}$$

where s denotes the recipient, t denotes the allocator, selfish behavior$_{st}$ is a measure of the number of tokens kept by the allocator, and trust flow$_{st}$ is the flow measure of trust computed from the social network for some value of K. The error term ε_{st} captures variation in trusting behavior that is not related to enforcement, including altruism and information effects.

Below, we provide evidence that estimating equation (6) in ordinary least squares yields $\beta < 0$ as predicted by our theory. One difficulty in interpreting the OLS β estimate is that the error term ε_{st} may be correlated with trust flow. This might happen because the network structure is endogenous to the pair of agents s and t, and also if other mechanisms for trusting behavior, such as altruism and information, operate more effectively for individuals who are socially closer. We try to address these endogeneity problems by flexibly controlling for demographics and various measures of social distance to pick up other mechanisms that can generate trust. However, given these endogeneity problems, we view the results mainly as evidence that flow measures of trust are related to trusting behavior in practice, and not a formal test.

Data description. In December 2003, subjects from two Harvard houses were recruited and asked to provide information on their social network as well as basic demographic data.[25] In May 2004 various dictator game treatments were run on these subjects. Within each house an equal number of player 1's and player 2's were selected, with player 1's having the role of the allocator.

[25] The data appendix describes the data collection procedure and experimental design in greater detail.

Each player 1 was matched with 5 potential player 2's: a direct friend, an indirect friend, a friend of an indirect friend, a student in the same staircase/floor who is at least distance 4 removed from the student, and a randomly selected student from the same house who falls in none of the above categories. In the dictator game, player 1 was asked to allocate 50 tokens between himself and player 2. In the treatments we use below, players were aware of each other's identity and actions, and each token was worth 1 point to the allocator and 3 points to the recipient. One point equaled 10 cents for both players, and hence the maximum winnings of a player in a match were $15.

Results. Using the self-reported social network data, we computed the trust measures implied by our model for small values of K. We made two assumptions in performing these calculations: 1) all links reported by either one of the two parties represent an actual social connection; 2) all links in the social network have equal capacity, normalized to one.

Figure 4 summarizes graphical evidence on the relationship (6). Following Andreoni and Miller (2002), in this figure we measure selfish behavior using a binary variable which equals 1 if the allocator chose to keep all tokens to himself, and zero otherwise. The four panels in the figure show the relationship between selfish behavior and trust flow computed for different values of K. For example, panel (a) uses our trust measure for $K = 0.5$. This measure equals 1 if the two players are connected and zero otherwise, hence the figure shows only two points, the average of selfish behavior for pairs of individuals where the trust flow equals 0, i.e., non-friends, and for pairs where the flow equals 1, i.e., friends. The other three panels are constructed analogously for $K = 1$, $K = 1.5$ and $K = 2$. All four panels in the figure confirm the prediction of the model by documenting a negative relationship between social collateral and selfish behavior.[26]

We evaluate the robustness of the graphical results by estimating a set of regressions augmented with other measures of social distance. We use fixed effects to control for (1) distance in the social network (length of shortest path) between the two players; (2) the year (freshmen, sophomore, junior, senior); (3) sex; and (4) staircase or floor of the allocator and the recipient. In addition, we include variables indicating whether the two players are in the same year and whether they live in the same staircase/floor. Table 2 reports the results. In columns 1-3, we use the binary indicator of selfish behavior as our dependent variable. Since our trust measure for $K = 0.5$ is collinear with the fixed effect for social distance of one, we use the trust flow for $K = 1$, $K = 1.5$ and $K = 2$ in these columns. Column 1 shows that the trust flow for $K = 1$ has the wrong sign and is no

[26] Estimating equation (6) in simple specifications with no controls (not reported) leads to a slope coefficient β that is negative and significant at the 5% level in all four cases.

longer significant, suggesting that this trust measure, which captures the number of common friends between allocator and recipient (see Proposition 2), does not contain much additional information beyond social distance. However, columns 2 and 3 show that trust flow for higher values of K predict trust even after controlling social distance. The effect of trust flow for $K = 1.5$ is significant at the 10% level (p-value equals 5.8%) while the effect of the trust flow for $K = 2$ is significant at 5% (p-value of 2.9%).

These patterns are also confirmed in columns 4-6, where we use the number of tokens kept by the allocator as the dependent variable. As in column 1, the trust flow for $K = 1$ is insignificant. However, columns 5 and 6 show that the trust measures for $K = 1.5$ and $K = 2$ have a negative effect on selfish behavior which are both significant at the 5% level (p-values are 2.2% and 0.4%). The effects documented here are not small. A one standard-deviation increase in trust for $K = 2$ reduces the amount kept by the allocator by about 52 cents and increases the amount that goes to the recipient by $1.56.

To summarize, there is a strong and robust relationship between network flow measures of trust and trusting behavior in practice, and our measures capture variation in trust that simple proxies for social distance do not account for. Developing more rigorous empirical applications of our model would be an interesting direction for further research.

5 Conclusion

This paper has built a model where agents use their social connections to secure informal loans. This function of social connections can be interpreted as one aspect of social capital, which we call social collateral. Our model provides a "rational" theory of trust in social networks and is therefore complementary to the growing experimental literature on trust which looks at trust between strangers (Berg, Dickhaut, and McCabe 1995). In recent years that literature has examined how trust varies with parties' background, including incomes, ethnic background and nationality (Glaeser, Laibson, Scheinkman, and Soutter 2000, Fershtman and Gneezy 2001). We hope to move in a similar direction and explore how our model explains differences in trusting behavior as a function of network structure in real-world social networks. A second interesting direction for research is to study the interaction between formal and informal contract enforcement, like in Kranton (1996) and Dixit (2003).

Appendix A: Proofs

Definition 6 *A weak flow with origin s is a function $g : W \times W \to \mathbb{R}$ with the properties*
(i) Skew symmetry: $g(u,v) = -g(v,u)$.
(ii) Capacity constraint: $g(u,v) \leq c(u,v)$.
(iii) Weak flow conservation: $\sum_w g(u,w) \leq 0$ unless $u = s$.

A weak flow of origin s can be thought of as taking a certain amount from node s, and carrying it to various other nodes in the network. By weak flow conservation, any node other than s receives a non-negative amount.

Lemma 1 *We can decompose any weak flow g as*

$$g = \sum_{u \in V} f_u$$

where for each u, f_u is an $s \to u$ flow, i.e., $\sum_w f_u(v,w) = 0$ for all $v \neq u$, $v \neq s$, and moreover $\sum_w f_u(u,w) = \sum_w g(u,w)$ i.e., f_u delivers the same amount to u that g does.

Proof. Consider vertex u such that $\sum_w g(u,w) < 0$. By weak flow conservation, the amount of the flow that is left at u must be coming from s. Hence there must be a flow $f_u \leq g$ carrying this amount from s. With f_u defined in such a way, repeat the same procedure for the weak flow $g - f_u$ with some other vertex u'. After defining f_u for all vertices u, the remainder f' satisfies flow conservation everywhere and can be added to any of the flows. ■

Implicit summation notation: For a weak flow g and two vertex sets $U \subseteq W$ and $V \subseteq W$, we use the notation that

$$f(U,V) = \sum_{u \in U,\ v \in V} f(u,v).$$

Proof of Theorem 1

Sufficiency. We begin by showing that when (4) holds, a side-deal proof equilibrium exists. By assumption, there exists an $s \to t$ flow with value V. For all u and v, let $h(u,v)$ equal the value assigned by this flow to the (u,v) link. Now consider the strategy profile where (1) the borrowing arrangement h is proposed and accepted; (2) the borrower returns the asset; (3) all transfers are paid if the borrower fails to return the asset. This strategy is clearly an equilibrium. To verify that it is side-deal proof, consider any side-deal, and let S denote the set of agents involved. For s to be strictly better off, it must be that he prefers not returning the asset in the side-deal. Now consider the (S,T) cut. By definition, the amount that flows through this cut under the original arrangement is V; but then the same amount must flow through the cut in the side-deal as well. This means that s must transfer at least V in the side-deal, but then he cannot be better off. More generally, this argument shows that any transfer arrangement that satisfies flow conservation is side-deal proof.

Necessity. We now show that when (4) is violated, no side-deal proof equilibrium exists. We proceed by assuming to the contrary that a pure strategy side-deal proof equilibrium implements borrowing even though (4) fails. First note that on the equilibrium path, the borrower must weakly prefer not to default. To see why, suppose that the borrower chooses to default on the equilibrium path. Since the lender and all intermediate agents must at least break even, this implies that the borrower has to make a transfer payment of at least V. But then the borrower must weakly prefer not to default, since returning the asset directly has a cost of V. This also implies that all intermediate agents must have a zero payoff.

By assumption, there exists an (S,T) cut with value $c(S,T) < V$. We now construct a side-deal where all intermediate agents in S continue to get zero, but the payoff of s strictly increases. The idea is easiest to understand in an equilibrium where promises are kept, i.e., when all transfers satisfy the capacity constraint $h(u,v) \leq c(u,v)$. Then, we simply construct an arrangement that satisfies flow conservation inside S, and delivers to the "boundary" of S the exact amount that was promised to be carried over to T under h. More generally, when the capacity constraints fail over some links, the deviation in the side-deal can result in some agents in S losing friendships with agents outside S. To compensate for this loss, the side-deal must deliver to the "boundary" of S an additional amount that equals the lost friendship value.

Formally, let g be a maximal $s \to t$ flow, and consider the restriction of g to S. This is a weak flow, and by the lemma it can be decomposed as $g = \sum_{u \in S} g_u$, where each g_u is an $s \to u$ flow. Now for each $u \in S$, let $g(u,T)$ and $h(u,T)$ denote the amounts leaving S through u under g and h. Moreover, for each $u \in S$, let $z(u,T)$ denote the total friendship value lost to u in the subgame where the borrower defaults, as a consequence of unkept transfer promises. Since g is a maximum flow and (S,T) is a minimal cut, it follows that $g(u,T) \geq h(u,T) + z(u,T)$. This is because any link between u and T is either represented in $h(u,T)$, if u pays the transfer, or $z(u,T)$, if u does not pay and loses the friendship. This inequality implies that, whenever $h(u,T) + z(u,T) > 0$, we also have $g(u,T) > 0$. As a result, we can define

$$h' = \sum_{u \in S} \frac{h(u,T) + z(u,T)}{g(u,T)} \cdot g_u.$$

Note that h' is a weak flow in S, and delivers exactly $h(u,T) + z(u,T)$ to all agents in S. Thus h' satisfies flow conservation within S and delivers to the "boundary" of S the sum of two terms: $h(u,T)$, which is the precise amount to be carried over to T under h; and $z(u,T)$ which is the loss of friendship u suffers due to not making other promised transfers. We claim that h' is a profitable side-deal. First, h' satisfies all capacity constraints by construction. Second, all agents in S break even under h', as they did in the original equilibrium. Third, the total value delivered by h is at most $c(S,T) < V$, which means that s pays less than V under h', while he pays exactly V in the original equilibrium. We have constructed a side-deal in which the borrower is better off and all other players are best-responding; hence the original equilibrium was not side-deal proof.

Proofs for Section 2.6

Multiple borrowers and lenders. We need to define the transfer arrangement and equilibrium selection in this environment. With multiple borrowers, agents have to worry about what happens when only a subset of borrowers default. To formalize this, assume that transfer payments can be made contingent on the set of borrowers who default. We continue to use side-deal proofness as our selection criterion, but extend the concept of a side-deal by assuming that the deviating group S must contain at least one borrower, and that at least one borrower in the side-deal is strictly better off.

Consider the directed network G' defined in the text. Note that in a directed network, the capacity map assigns values to directed links; this means in particular that while $c(s_0, s_i) > 0$, we have $c(s_i, s_0) = 0$, because there is no $s_i \rightarrow s_0$ link. The concept of a network flow can be straightforwardly extended to directed networks. Denote the maximum $s_0 \rightarrow t_0$ flow in G' by $T^{s_0 t_0}(c)$.

Proposition 4 *There exists a side-deal proof equilibrium in which the borrowing demands of all borrowers are satisfied if and only if*

$$V_1 + ... + V_k \leq T^{s_0 t_0}(c). \tag{7}$$

Proof. *Sufficiency.* Suppose that (7) holds, and let h be a maximum $s_0 \rightarrow t_0$ flow. Like in the basic version of the model, we use this flow to define an equilibrium transfer arrangement. The present setting has two additional complications: we need to define how much each borrower borrows from each lender, and to ensure enforcement of all borrowing contracts, we need to introduce transfer payments that can be contingent on the set of agents who default.

To deal with these problems, first note that cutting all links of s_0 in G' has value equal to $V_1 + ... + V_k$, and hence h can carry at most this amount. But then inequality (7) must hold with equality, and h must use all links originating in s_0 at full capacity. Now, by the argument of Lemma 1 we can decompose the restriction of h to G as

$$h = \sum_{i,j} h^{ij}$$

where h^{ij} is a $s_i \rightarrow t_j$ flow. Let $h^i = \sum_j h^{ij}$, then h^i is a flow with source s_i, that takes exactly V_i and distributes it in some way across the lenders t_j. Let $h^i(t_j)$ be the amount that h^i takes to t_j; this is the amount that in our equilibrium borrower i will ask from lender j. Note that this construction satisfies all resource constraints: $\sum_j h^i(t_j) = V_i$, thus the total amount that i borrows is exactly V_i; and $\sum_i h^i(t_j) \leq c(t_j, t_0) = W_j$, i.e., the total amount borrowed from j does not exceed his resources W_j.

The h^{ij} decomposition can also be used to define the state-contingent transfer payments. More specifically, if agent i is the only person who defaults, then let the transfer payments be defined by the flow h^i. If more than one agent chooses to default, then define transfers to be the sum of h^i for all i who defaulted. This completes the definition of our candidate side-deal proof equilibrium. Showing that this candidate is really an equilibrium admitting no side-deals is completely analogous to the proof of Theorem 1.

Necessity. Suppose that (7) fails, and let (S, T) be a minimum cut. Because the total value of all links originating in s_0 is $V_1 + ... + V_k$ and the minimum cut is less than this amount, the intersection of S and the set of borrowers $\{s_1, ..., s_k\}$ is non-empty. By relabelling agents if necessary, we can assume that borrowers s_1,...,s_l are in S, while borrowers s_{l+1},...,s_k are in T. This means that the links between s_0 and s_{l+1},...,s_k have all been cut. The value of this part of the (S, T) cut is clearly $V_{l+1} + ... + V_k$. Now consider the restriction of the (S, T) cut to the graph G. In this restriction, we are excluding all cut links between s_0 and s_{l+1},...,s_k, among others. As a result, the value of the restriction of the cut to G must be less than $(V_1 + ... + V_k) - (V_{l+1} + ... + V_k) = V_1 + ... + V_l$.

The proof is completed by constructing a side-deal in which agents in S deviate. The idea is simple: the total obligation of agents in S is if borrowers s_1,...,s_k default is limited by the value of the (S, T) cut restricted to G. But we have just seen that this value is strictly less than $V_1 + ... + V_l$, whereas joint default by s_1,...,s_l yields a profit of exactly $V_1 + ... + V_l$. It follows that agents in S as a group do not have the right incentives to repay the loans. The actual construction of the side-deal is very similar to the construction used in the proof of Theorem 1, and is therefore omitted. ■

Transfer constraints. In this analysis, we use a more stringent equilibrium selection criterion: We look for equilibria where (i) all promised transfers are paid; (ii) there are no profitable side-deals. In the earlier analysis, there was no need to impose (i), because the characterization results showed that any level of borrowing that can be implemented can also be implemented using equilibria where all transfers are paid. With transfer constraints, requiring that all promises are credible has additional bite, because promises that are not credible can generate large punishment in the form of loss of friendship to agents who have small k_u. We find it plausible that such agents will not make promises that they know they cannot keep; but instead of providing formal microfoundations for this, we simply restrict ourselves to equilibria that are "credible" in the sense that all promises are kept.

Consider the directed network G'' defined in the text, and let the maximum $s_1 \to t_1$ flow in G'' be denoted by $T_K^{s_1 t_1}(c)$.

Proposition 5 *There exists a side-deal proof equilibrium with credible promises that implements borrowing if and only if*

$$V \leq T_K^{s_1 t_1}(c). \tag{8}$$

Proof. *Sufficiency.* If (8) holds, then take a flow with value V, and let the flow values between different agents define the transfer arrangement in our candidate equilibrium. Note that by construction, this borrowing arrangement satisfies the borrowing constraints of all agents u. Moreover, the promised transfers in this arrangement will be kept because they all satisfy the capacity constraint. It remains to show that there are no profitable side-deals; this follows from the same argument used in the proof of Theorem 1.

Necessity. Suppose that (8) fails, and consider an equilibrium where promised transfers are paid and borrowing is implemented. We now show that this equilibrium admits a side-deal. Our argument is similar to the proof of Theorem 1 in that we build the side-deal using a minimum cut on the network G'. However, the present setup has one additional difficulty: we need to make sure that the side-deal emerging from the minimum cut does not separate agents from their duplicates.

Let (S', T') be a minimum cut. If for some $u \neq s$ we have $u_2 \in S'$, then $u_1 \in S'$ also holds, because u_2 has only one incoming link, which originates in u_1. Let S be the union of s and the collection of agents u such that $u_1 \in S'$. We need to show that agents in S as a group do not have the right incentive to return the asset. To see why, consider first an agent $u \in S$ such that $u_2 \notin S'$. It follows that the (S', T') cut separated u_1 from u_2, by cutting the $u_1 \to u_2$ link. But in this equilibrium, promises are kept, and hence the total obligation of u to agents outside S can be at most k_u, which is exactly the value of the cut link. Next consider an agent $u \in S$ such that $u_2 \in S$'. For this agent, the total obligations to others outside S are bounded from above by the total value of the links originating in u_2 that are cut. Summing over all $u \in S$, we conclude that the total obligation of all agents in S do not exceed the value of the (S', T') cut, and hence is strictly smaller than V. Thus S as a group has an incentive to default. The actual side-deal can now be constructed in the same way as in the proof of Theorem 1. ■

Proof of Proposition 1

(i) Consider two capacities $c_1 \leq c_2$. Any K-flow between s and t that is feasible under c_1 is also feasible under c_2; hence the maximum K-flow cannot be lower under c_2 than under c_1.

(ii) Consider two capacities c_1 and c_2, and let f_1 and f_2 be maximum K-flows under c_1 and c_2. Let $c_3 = \alpha c_1 + (1 - \alpha) c_2$ where $0 \leq \alpha \leq 1$. Clearly c_3 is a capacity. Consider $f_3 = \alpha f_1 + (1 - \alpha) f_2$. We argue that f_3 is a feasible K-flow under c_3. First, f_3 is a flow: skew symmetry and flow conservation are preserved by convex combinations, and the capacity constraint $f_3 \leq c_3$ will be satisfied by construction. Second, since both f_1 and f_2 use edges that are within distance K from s, the same holds for f_3. The value of f_3 is $|f_3| = \alpha |f_1| + (1-\alpha)|f_2| = \alpha T_K^{st}(c_1) + (1-\alpha) T_K^{st}(c_2)$. Finally, $|f_3| \leq T_K^{st}(c_3)$ by definition, showing that T_K^{ij} is a concave function.

(iii) Consider $K_1 \leq K_2$. Any K_1-flow f_1 is also a K_2-flow, hence the maximum K_2-flow cannot be lower than the maximum K_1-flow.

Proof of Proposition 2

(i) If $K = 0.5$ then the only edges that are at most K from s are those between s and some $t \in N_s$. For all such t, we have $T_K^{st}(c) = 1$, hence $T_K^s(c) = |N_s|$ and $T_K(c) = 2|E|$.

(ii) Here, the set of edges that are at most K away from s include those connecting neighbors of s. As a result, for any $t \in N_s$, $T_K^{st}(c)$ equals 1 plus the number of links between t and other vertices in N_s. Thus $T_K^s(c)$ is $|N_s|$ plus twice the number of links connecting vertices in N_s, which can also be written as $|N_s|(1 + (|N_s| - 1)\kappa_s)$. Summing over s, we count all edges twice, plus all edges that are part of a triangle two more times, for a total of $2|E| + 6 \cdot \Delta$.

(iii) Suppose first that the distance between s and t is two. For any feasible $s \to t$ path, consider the final link $u \to t$: since $K = 1.5$, it must be that $u \in N_s$. As a result, there exists a one-to-one correspondence between feasible $s \to t$ paths and (u,t) links such that $u \in N_s$, and the number of such links equals n_{st}. Next suppose that $t \in N_s$. Now in addition to $s \to t$ paths where the final link $u \to t$ is such that $u \in N_s$, we also have the possibility that $u = s$, and that u is at distance 2 from s. In the latter case, consider the second-to-last link in the path: $v \to u$. As before, we must have $v \in N_s$. Hence every time there is a feasible $s \to t$ flow where the last link runs from some agent $u \notin N_s$, agent u must be connected to s on a second path of length 2 (i.e., $s \to v \to u$). The number of such u agents is $\sum_{u \in N_s} I_{ut}^s$.

Finally to compute $T_K^s(c)$, we need to add up $T_K^{st}(c)$ for all t. Adding up the terms $n_{st} + 1_{\{t \in N_s\}}$ is easily seen to give $\sum_{t' \in N_s} |N_{t'}|$. Summing $1_{\{t \in N_s\}} \cdot \sum_{u \in N_s} I_{ut}^s$ gives $\sum_{u,v \in N_s} I_{uv}^s$.

Proof of Proposition 3

The expected payoff of agent s conditional on him being the borrower can be written as

$$\frac{1}{N} \sum_j \frac{q_s(j)}{j} \Pi(j) = \frac{T_K^s(c)}{N} \sum_j \frac{q_s(j)}{T_K^s(c)} \cdot \frac{\Pi(j)}{j}$$

which can be viewed as the expected value of the function $\Pi(j)/j$ under the probability density $q_s(j)/T_K^s(c)$. When technology favors high asset values, $\Pi(v)$ is convex; this, combined with the fact that $\Pi(0) = 0$ implies that $\Pi(v)/v$ is nondecreasing. In this case, a first-order stochastic dominance increase in the probability density $q_s(j)/T_K^s(c)$ increases the expected payoff by definition. An analogous argument shows that when technology favors low asset values, the same increase in the sense of first-order stochastic dominance reduces the expected payoff of s.

Appendix B: Microfoundations for social sanctions

In this section we develop a model where punishment at the level of the link arises endogenously. There are three key changes relative to the model presented in the main text: (1) with probability $p > 0$, the asset disappears – e.g., stolen by a third party – after the borrower uses it. (2) Each link "goes bad" with a small probability ε during the model, capturing the idea that friendships can disappear for exogenous reasons. (3) The utility of friendship is modelled using a "friendship game" where agents can choose to interact or stay away from each other. The payoffs of this friendship game depend on the capacity of the link and on whether the link has gone bad.

Model setup. This model consists of six stages, which are the following:

Stage 1: Realization of needs. Identical to stage 1 in Section 2.

Stage 2: Borrowing arrangement. In this model, there is uncertainty about whether the asset disappears after being used. As a result, an arrangement is now a set of state contingent payments, where the publicly observable state of the world i is either $i = 0$, if the asset is returned, or $i = 1$ if the asset is reported stolen. A borrowing agreement consists of two parts. 1) A contract specifying payments y_i to be made by the borrower to the lender in the two states ($i = 0$ or 1). This contract can be thought of as a traditional incentive contract to solve the moral hazard problem in lending. If there was a perfect court system in the economy, then this contract would be sufficient to achieve efficient lending. 2) A *transfer arrangement* specifying payments $h_i(u, v)$ to be made between agents in the social network if the borrower fails to make the payment y_i. Here $h_i(u, v)$ denotes a payment to be made by u to v in state i. The links for which transfers can be proposed must be within distance K from the borrower.

Stage 3: Repayment. If an arrangement was reached in stage 2, the asset is borrowed and s earns an income of $\omega(V)$, where $\omega(.)$ is a differentiable, non-decreasing function. Following the use of the asset, with probability p it is stolen. We assume that $\omega(V) > pV$ for all V in the support of F, which guarantees that lending the asset is the socially efficient allocation. Even if the asset is not stolen, the borrower may choose to pretend that it is stolen, and sell it at the liquidation value of $\phi \cdot V$ where $\phi < 1$. The borrower then chooses whether to make the payment y_i specified in the contract.

Stage 4: Bad links. At this stage, any link in the network may "go bad" with some small probability. We think of bad links as the realization by a player that he no longer requires the business or friendship services of his friend. As we describe below, cooperation over bad links in the friendship game is no longer beneficial. Therefore agents who learn that a link has gone bad will find it optimal not to make a promised transfer along the link. From a technical perspective, bad links are a tool to generate cooperation without repeated play, just like the "Machiavellian types" in Dixit (2003) (see also Benoit and Krishna (1985)). In an equilibrium where promised transfers are expected to be paid, failure by u to make a payment will be interpreted by v as evidence that the link has gone bad. In this case, v will defect in the friendship phase, which reduces the payoff of the deviator u by $c(u, v)$.

To formalize bad links, assume that for every link of every agent, with a small probability $\varepsilon > 0$ independent across agents and links, the player learns that his link has gone bad at this stage. Thus, for any link (u, v), the probability that the link has not gone bad is $(1-\varepsilon)^2$; and for any link (u, v) where u does not learn that the link has gone bad, u still believes, correctly, that with probability ε the link has gone bad.

Stage 5: Transfer payments. If the borrower chose to make the payment y_i in stage 3, then this stage of the game is skipped, and play moves on to the friendship phase. If the borrower did not make the payment y_i, then at this stage agents in the social network choose whether to make the prescribed transfers $h_i(u, v)$. Each agent has a binary choice: either he makes the promised payment in full or he pays nothing.

Stage 6: Friendship game. Each link between two agents u and v has a *friendship game* with an associated value $c(u, v)$. As long as the link is good, the friendship game is a two-player coordination game with two actions, with payoffs as depicted below.

	C	D
C	$c(u,v)$ $c(u,v)$	0 $c(u,v)/2$
D	$c(u,v)/2$ 0	-1 -1

This game has a unique equilibrium (C,C) with payoff $c(u,v)$ to both parties, which represents the benefit from friendly interactions. A party only derives positive benefits if his friend chooses to cooperate; and benefits are highest when there is mutual cooperation. If a link has gone bad, cooperation is no longer beneficial, and the payoffs of the friendship game change as follows.

	C	D
C	-1 -1	0 0
D	0 0	0 0

Here, mutual cooperation leads to the low payoff of -1, capturing the idea that parties who are no longer friends might find it unpleasant to interact. If either party defects, the payoff of both parties is set to zero. The payoffs in the friendship game imply that if a player knows that a link has gone bad with probability 1, a best response is to play D.

Model analysis. Because there is uncertainty in this model, we need to extend the concept of side-deals to Bayesian games.

Definition 7 *Consider a pure strategy profile σ and a set of beliefs μ. A side-deal with respect to (σ,μ) is a set of agents S, a transfer arrangement $\widetilde{h}_i(u,v)$ for all $u,v \in S$, and a set of continuation strategies and beliefs $\{(\widetilde{\sigma}_u,\widetilde{\mu}_u)\,|u\in S\}$ proposed by s to agents in at the end of stage 2, such that*
(i) $U_u(\widetilde{\sigma}_u,\widetilde{\sigma}_{S-u},\sigma_{-S}\mid\widetilde{\mu}_u)\geq U_u(\sigma'_u,\widetilde{\sigma}_{S-u},\sigma_{-S}\mid\widetilde{\mu}_u)$ for all σ'_u and all $u\in S$,
(ii) The beliefs $\widetilde{\mu}$ satisfy Bayes rule whenever possible if play is determined by $(\widetilde{\sigma}_S,\sigma_{-S})$,
(iii) $U_u(\widetilde{\sigma}_S,\sigma_{-S}\mid\widetilde{\mu}_u)\geq U_u(\sigma_S,\sigma_{-S}\mid\mu)$ for all $u\in S$,
(iv) $U_s(\widetilde{\sigma}_S,\sigma_{-S}\mid\widetilde{\mu}_u)>U_s(\sigma_S,\sigma_{-S}\mid\mu)$.

The only conceptually new condition is (ii), which is clearly needed in a Bayesian environment. Motivated by this definition, our equilibrium concept will be side-deal proof perfect Bayesian equilibrium.

Theorem 2 *There exists a side-deal proof perfect Bayesian equilibrium that implements borrowing between s and t if and only if the asset value V satisfies*

$$V \leq T_K^{st}(c)\cdot\frac{(1-\varepsilon)^2}{\phi+p(1-\phi)}. \tag{9}$$

Proof. We begin by analyzing the optimal incentive contract in the absence of enforcement constraints. Suppose that s makes payments x_i ($i=0$ or $i=1$) in the two states of the world. What values of x_i guarantee that s chooses to return the asset while t breaks even? To prevent s from stealing, the excess payment if the asset is reported stolen must exceed the liquidation value ϕV:

$$x_1 - x_0 \geq \phi V. \tag{10}$$

In order for the lender to break even, he has to receive at least pV in expectation:

$$px_1 + (1-p)\,x_0 \geq pV. \tag{11}$$

The minimum transfers which satisfy (10) and (11) are

$$x_0 = p(1-\phi)V \quad\text{and}\quad x_1 = [\phi + p(1-\phi)]\,V. \tag{12}$$

Bringing back the enforcement constraints, it is intuitive that borrowing can be implemented in the network as long as $\max[x_0, x_1]$ does not exceed the maximum flow between s and t: in that case, the lender can just transfer x_i to the borrower along the network. Since $x_1 > x_0$, this requires that x_1 does not exceed the maximum flow, or equivalently

$$V \leq c(s,t) \cdot \frac{(1-\varepsilon)^2}{\phi + p(1-\phi)}$$

which is indeed the condition in the theorem. We now turn to the proof.

Sufficiency. We begin by showing that when (9) holds, a side-deal proof equilibrium exists. Let x_i be defined by (12) and let $y_i = x_i$. By assumption, there exists a flow with respect to the capacity c that carries $x_1/(1-\varepsilon)^2$ from s to t. For all u and v, define $h_1(u,v)$ to be $1-\varepsilon$ times the value assigned by this flow to the (u,v) link. Similarly, let $h_0(u,v)$ be equal to $1-\varepsilon$ times a flow that carries $x_0/(1-\varepsilon)^2$ from s to t. Now consider the strategy profile in which (1) the transfer arrangement (x_i, h_i) is proposed and accepted, (2) the asset is borrowed and returned unless stolen, (3) every agent u pays every promised transfer $h_i(u,v)$ if necessary, unless he learns that his link with v has gone bad, (4) all agents play C in the friendship game unless they learn that the link has gone bad, in which case they play D. This strategy profile σ generates beliefs μ, and (σ, μ) constitute a perfect Bayesian equilibrium. To see why, note that conditional on others making the transfer payments, it is optimal for s to make the payments y_i and not to steal the asset. Also, since $h_i(u,v) \leq (1-\varepsilon)c(u,v)$, all agents find it optimal to make the transfer payments given beliefs. Finally, because on path play never gets to the transfers, all intermediate agents are indifferent between accepting the deal and rejecting it. In fact, even if the transfers were used in one or both states on path, intermediate agents would still break even, because h_i are defined using flows.

We also need to verify that the equilibrium proposed here is side-deal proof. Consider any side-deal, and let S denote the set of agents involved. Suppose that after the side-deal, the borrower reports that the asset is stolen with probability $p' \geq p$. Let T be the complement of S in W, and consider the (S,T) cut. By definition, the expected amount that flows through the (S,T) cut in state i if y_i is not paid equals x_i. If the borrower never chooses to pay y_i in the side-deal, he will have to make sure that at least $p'x_1 + (1-p')x_0$ gets to the cut in expectation. Because all intermediate agents must break even in expectation, this implies that s's expected payments must be $p'x_1 + (1-p')x_0$ or more. Thus the side-deal comes with a cost increase of $(p'-p)[x_1 - x_0]$. The increase in expected cost is easily seen to be the same if the borrower chooses to pay y_i in one or both states. The expected benefit of the side-deal is $(p'-p)\phi V$. By equation (10) the expected benefit does not exceed the expected cost; the side-deal is not profitable to s, which is a contradiction. Hence the original arrangement was side-deal proof.

Necessity. We now show that when (9) is violated, no side-deal proof equilibrium exists. We proceed by assuming to the contrary that a pure strategy side-deal proof perfect Bayesian equilibrium implements borrowing even though (9) fails. For simplicity, we assume that the equilibrium proposed transfers $h_i(u,v)$ are expected to be paid by all agents u in stage 5 if the borrower chooses not to pay y_i directly; i.e., we only focus on equilibria where promises are kept. This condition is not necessary to obtain the result, but simplifies the proof somewhat. If this condition holds, then $h_i(u,v) \leq (1-\varepsilon)c(u,v)$ holds for all transfers proposed in equilibrium, because the amount that u can expect to benefit from his friendship with v is at most $(1-\varepsilon)c(u,v)$.

Let $\chi_i = 1$ if in state i on the equilibrium path, s chooses not to pay y_i, and let $\chi_i = 0$ otherwise.

Case I: $\chi_0 = \chi_1 = 1$.

In this case, on the equilibrium path, y_i are never paid, and instead the transfer arrangements are always used. Define the expected transfer $h = ph_1 + (1-p)h_0$. By the individual rationality of intermediate agents, h satisfies weak flow conservation, and therefore by the Lemma can be decomposed as

$$h = \sum_{u \in V,\ u \neq t} f_u + h'$$

where f_u is $s \to u$ flow and $h' = f_t$. In words, the f_u flows deliver the expected profits to the intermediate agents, while h' is an $s \to t$ flow that delivers the expected payoff to the lender. Denote $\sum_{u \neq t} f_u = f$, then f is a weak flow delivering the payments to all intermediate agents.

Our proof strategy will be the following. First, we take out the profits of all intermediate agents from the capacity c and the transfer h, essentially creating a "reduced" problem where intermediate agents are expected to break even. Then we construct a side-deal for this simpler case using the maximum flow minimum cut theorem, and finally transform this into a side-deal of the original setup.

Let $c'(u,v) = c(u,v) - f(u,v)/(1-\varepsilon)$ be a capacity on G. Note that any flow g' under c' can be transformed into a flow $g = g' + f/(1-\varepsilon)$ that satisfies the capacity constraints c. Consider the functions $h'_i = h_i - f$. It is easy to verify that $h'_i/(1-\varepsilon)$ satisfy the capacity constraints with respect to c', and that $h' = ph'_1 + (1-p)h'_0$. Let (S,T) be a minimal cut of the directed flow network with capacity c'. By the maximum flow-minimum cut theorem, there exists a maximum flow g in the network that uses the full capacity of this cut. By assumption, the value of the cut under h'_1 satisfies $h'_1(S,T)/(1-\varepsilon) \leq g(S,T) < x_1/(1-\varepsilon)^2$, which implies that $(1-\varepsilon)[h'_1(S,T) - h'_0(S,T)] < \phi V$ because $(1-\varepsilon)|h| \geq pV$. In words, the value flowing through the minimal cut in the two states does not provide sufficient incentives to not steal the asset.

We now construct a side-deal for the reduced problem. The idea is to construct a transfer arrangement that satisfies flow conservation inside S, and delivers to the "boundary" of S the exact amount that was promised to be carried over to T under h'. With such an arrangement, all agents in S will break even in each state, and thus the incentives that applied to S as a group will apply directly to agent s. Since S as a group did not have the right incentives, with the side-deal s will not have the right incentives either.

Formally, using the implicit summation notation, let for each $u \in S$, $g(u,T)$, $h'_1(u,T)$ and $h'_0(u,T)$ denote the amounts leaving S through u via the maximum flow g, h'_1, and h'_0. Clearly, $(1-\varepsilon)g(u,T) \geq h'_1(u,T)$ and $(1-\varepsilon)g(u,T) \geq h'_0(u,T)$. Now consider the restriction of g to the set S. This is a weak flow, and by the lemma it can be decomposed as $g = \sum_{u \in S} g_u$. Define $h''_1 = \sum_{u \in S}(h'_1(u,T)/g(u,T)) \cdot g_u$ and $h''_0 = \sum_{u \in S}(h'_0(u,T)/g(u,T)) \cdot g_u$. Then h''_1 and h''_0 are both weak flows in S, they satisfy $h''_i \leq (1-\varepsilon)c'$, and deliver exactly $h'_1(u)$ and $h'_0(u)$ to all $u \in S$. Thus h''_i satisfies flow conservation within S, and delivers to the "boundary" of S the amount promised to be carried over to T under h'_1, as desired. The total value delivered by h''_i is the value of the cut links under h'_i; hence the amount that leaves s in the two states under h'' satisfies $(1-\varepsilon)[|h''_1| - |h''_0|] < x_1 - x_0$, i.e., is insufficient to provide incentives not to steal the asset.

Now go back to the original network, and consider a side-deal with all agents in the set S, where these agents are promised a transfer arrangement $f + h''_i$. This is just adding back the profits of all agents to the side-deal of the reduced problem. With this definition, the new side-deal satisfies the capacity constraints $f + h''_i \leq (1-\varepsilon)c$ because $h''_i \leq (1-\varepsilon)c' = (1-\varepsilon)c - f$. Second, all agents in S will be indifferent, because they get the same expected profits delivered by f (note that h'' is a flow in both states and thus nets to zero state by state). The agents who have links that are in the cut are indifferent because h'' is defined such that its inflow equals the required outflow for these agents. Third, the side-deal does not have enough incentives for s not to steal the asset, because $|h''_1| - |h''_0| < \phi V/(1-\varepsilon)$. Moreover, if the original deal was beneficial for s, then so is the new deal. This is because the cost of the original deal was $|f| + |h'|$. The cost of the new deal if the borrower follows the honest asset-return policy is $|f|+|h''|$. But both h' and h'' are flows, and they are equal on the (S,T) cut, hence they have equal values. Therefore by following an honest policy, the borrower will have a cost equal to what he had to pay in the original deal. However, since the incentive compatibility constraint is not satisfied, the borrower is strictly better off always stealing the asset in the side deal. This argument shows that there exists a side-deal in which the borrower is strictly better off, and all other players are best-responding; hence the original equilibrium was not side-deal proof.

It remains to consider the cases where either χ_0 or χ_1 is equal to zero. In these cases, define the expected transfer payments as $h = p\chi_1 h_1 + (1-p)\chi_0 h_0$. As above, h is a weak flow and thus f, the weak flow delivering the expected profits to all intermediate agents can be defined. Similarly, one can define c' and h'_i, and letting (S,T) be the minimal cut of c', $h'_1(S,T)/(1-\varepsilon) < x_1/(1-\varepsilon)^2$ must hold.

Case II: $\chi_0 = 1$ and $\chi_1 = 0$.

Then $h = (1-p)h_0$ and the decomposition $h = f + h'$ yields $h_0 = f/(1-p) + h'/(1-p)$ so that $h'_0 = h_0 - f = f \cdot p/(1-p) + h'/(1-p)$ is a weak flow, because it is a sum of two weak flows. It follows that $|h_0| = |f| + |h'_0| \geq |f| + |h'_0(S,T)|$. Therefore $|f| + |h''_0| \leq |h_0|$ because h''_0 is a flow and $h''_0 = h'_0$ on the (S,T) cut. Moreover, incentive compatibility requires $y_1 - (1-\varepsilon)|h_0| \geq \phi V$, while the break-even constraint of the lender means that $py_1 + (1-p)(1-\varepsilon)[|h_0| - |f|/(1-p)] \geq pV$. Combining these inequalities gives $y_1 \geq x_1 + (1-\varepsilon)|f|$. Now consider the side-deal $h''_i + f$ defined as above. Since $|h''_0 + f| \leq |h_0| \leq y_0/(1-\varepsilon)$ and $|h''_1 + f| < x_1/(1-\varepsilon) + |f| \leq y_1/(1-\varepsilon)$, the borrower will strictly prefer this arrangement to the previous one. Since all intermediate agents get net profits delivered by f in both states in the side-deal, they are indifferent. Thus the proposed arrangement is indeed a side-deal.

Case III: $\chi_0 = 0$ and $\chi_1 = 1$.

Here h_1 is a weak flow, which must deliver less than $x_1/(1-\varepsilon)$ to t because by assumption $x_1/(1-\varepsilon)$ is more than the maximum flow. Thus incentive compatibility fails with the original agreement; even without any side-deal, the lender is better off not returning the asset.

Case IV: $\chi_0 = 0$ and $\chi_1 = 0$.

Here a valid side-deal is to pay y_0 in state zero and propose the transfer arrangement h''_1 for state 1. All intermediate agents are indifferent since they were getting zero in the original arrangement, and because $h''_1 < x_1/(1-\varepsilon) \leq y_1/(1-\varepsilon)$ the expected payment in the side-deal is strictly lower than in the original deal.

In the proof so far, we only considered the case where the borrower does not steal the asset on the equilibrium path. If the equilibrium is such that the borrower always steals, then $\min[(1-\varepsilon)|h_1|, y_1] \geq V$ must hold. If $\chi_1 = 1$ then $h_1/(1-\varepsilon)$ is a weak flow with respect to capacity c that must transfer at least $V/(1-\varepsilon)^2$ to t. This leads to a condition on the maximum $s \to t$ flow that is stronger than (9). If $\chi_1 = 0$, then a valid side-deal is to propose the transfer arrangement h''_1 for both states. As above, all intermediate agents are indifferent, and $h''_1 < x_1/(1-\varepsilon) \leq y_1/(1-\varepsilon)$ holds which proves that the expected payment in the side-deal is strictly lower than in the original deal.

Appendix C: Data

Data for Table 1. Row 1 is based on the 1986 International Social Survey Programme module on Social Support and Networks of the GSS. Question 1018 asks "Suppose you need to borrow a large sum of money. A. Who would you turn to first? B. Who would you turn to second?" Possible answers are husband/wife/partner, mother, father, daughter, son, sister, brother, other relative including in-laws, closest friend, other friend, neighbor and someone you work with, which we classify as persons in the immediate social network of the respondent, as well as bank/credit union financial institution, Savings&loan, employer, government or social services, other, no one, don't know and no answer.

Row 2 uses the Micro and Small Enterprises Survey conducted by Ageba and Amha in Ethiopia in 2003. Enterprises were asked if they ever received credit from various sources; the proportion of firms who received credit from friends/relatives is reported. See Table 1 in Ageba and Amha (2006) for details.

Row 3 makes use of survey data collected by the Regional Program for Enterprise Development (RPED) of the World Bank for 224 manufacturing firms in Kenya in 1993. Firms were asked how they started their business; the proportion of firms who used loans from friends and relatives is reported. See Table 21 in Fafchamps, Biggs, Conning, and Srivastava (1994) for details.

Rows 4 and 5 use data from the 1996 topical module of the GSS on Markets. For example, purchasers of cars from individuals were asked "Which of the following best describes your relationship to the person who sold you the vehicle at the time of the purchase?" and purchasers of cars from dealers were asked about "your relationship to the salesperson from whom you purchased your car or to the owner of the auto dealership." Respondents were asked to choose from the following responses: a relative (including in-laws), a friend or acquaintance, a friend of a friend or relative or a relative of a friend, not a friend, but someone with whom I had previous business dealings, or no prior relationship. We classify all but the last of these responses as persons belonging to the respondent's immediate social network. Similar questions were asked for home purchases. See Table 1 in DiMaggio and Louch (1998) for details.

Row 6 uses data from the 1991 and 1992 Current Population Surveys. The CPS asks active jobseekers about their method of job search. Potential answers are: checked with public employment agency, checked with private employment agency, checked with employer directly, checked with friends or relatives, placed or answered ads, and used other search methods. We classify friends or relatives as belonging to the immediate social network of the jobseeker. See also Table 1 in Bortnick and Ports (1992) and also Ports (1993).

Row 7 is based on data from the 1993 Panel Study of Income Dynamics. The job search categories in the PSID are the same as in the CPS. See appendix table 1 in Ioannides and Loury (2004) for details.

Experimental Data for Section 4. In December 2003 Harvard undergraduate subjects were recruited through posters, flyers and mail invitation and directed to a website. Subjects provided their email address and were sent a password. Subjects without a valid email address were excluded. All future earnings from the experiment were transferred to the electronic cash-card account of the student.

Subjects who logged onto the website had to (1) report information about their social network and (2) fill in a questionnaire asking basic demographic information. Subjects were given incentive to report their friends truthfully: they received 50 cents with 50 percent probability if they named each other. The expected payoff of 25 cents was set to be sufficiently large to give subjects an incentive to report their friends truthfully but not large enough to induce 'gaming'. The randomization was included to help avoid disappointment if a subject is not named by enough friends. The technique of providing monetary incentives to truthfully reveal friendships worked well. The pilot focused on two houses with 806 students in total, of whom 569 signed up. The survey netted 5690 one-way links. Of those, 2086 links were symmetric links where both agents named each other. For symmetric links, the two parties' assessment of the time they spend together in a typical week was within half and hour in 80% of all cases. The social network was constructed as an OR-network where two subjects share a link if either of them named the other.

The total earnings of subjects in the pilot consisted of (1) a baseline compensation for completing the full online survey and (2) the earnings from the dictator games. Subjects also entered a raffle where they could win valuable prizes nine months later provided they completed the initial surveys plus all follow-up treatments.

Treatments. In May 2004 various treatments were run to measure social preferences of the students. Within each house an equal number of allocators (player 1) and recipients (player 2) were randomly selected. Allocators faced five different recipients in a modified dictator game as described in the text, playing each of them in two situations, once anonymously and once non-anonymously. In each pairing, the allocator made three decisions about sharing 50 tokens between himself and the recipient. In the first decision the token was worth 1 point to the allocator and 3 points to the recipient; in the second decision the tokens were worth 2 points to both players; in the third decision the tokens were worth 3 points to player 1 and 1 point to the other player. One point equalled 10 cents. All decisions, situations and pairs were randomly presented to each player. One of their decisions for one pair in one situation (anonymous or non-anonymous) was randomly selected and implemented. The design ensured that each recipient was matched up with exactly one allocator during this process.[27]

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[27] Allocators were invited a second time and matched with a random player from their house whose name they would not find out. We do not use these decisions in this paper because trust flows are undefined when the identity of the recipient is unknown to the allocator.

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TABLE I

The use of social networks in informal exchanges

Type of exchange	Country	Year	Proportion who rely on their social network
A. Borrowing			
1. A large sum of money Authors' calculations from GSS	United States	1995	55%
2. Small enterprise borrowing Ageba-Amha (2006)	Ethiopia	2003	27%
3. Startup capital for micro firms Fafchamps et al (1994)	Kenya	1993	9%
B. Purchases			
4. Home purchase DiMaggio-Louch (1998)	United States	1996	40%
5. Used car purchase DiMaggio-Louch (1998)	United States	1996	44%
C. Job search			
6. Unemployed Bortnick-Ports (1992), Ports (1993)	United States	1991-92	23%
7. On-the-job searchers Ioannides-Loury (2004)	United States	1993	8.5%

NOTE-- Table shows proportion of people who would or do rely on their immediate social network for various transactions. Immediate social network always includes relatives and friends; in some studies it also includes friends of relatives and friends of friends as well as business relationships. See the data appendix for details on each environment.

TABLE II

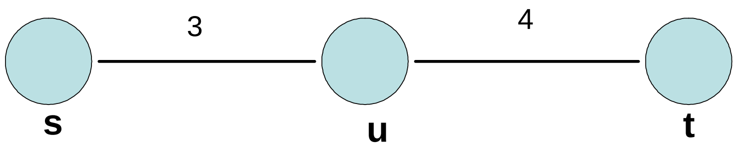
Trust flow and selfish behavior

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent var.:	**Indicator for selfish behavior**			**Tokens kept by allocator**		
Trust flow for K=1.0	**0.0124**			**0.285**		
	(0.0175)			(0.6052)		
Trust flow for K=1.5		**-0.0188**			**-0.922**	
		(0.0098)			(0.3986)	
Trust flow for K=2.0			**-0.0184**			**-1.004**
			(0.0083)			(0.3398)
Allocator sex	-0.0137	-0.0129	-0.0172	-3.459	-3.419	-3.646
	(0.0682)	(0.068)	(0.068)	(2.719)	(2.702)	(2.698)
Recipient sex	0.0721	0.076	0.0742	2.116	1.293	2.236
	(0.0332)	(0.0327)	(0.0327)	(1.318)	(1.316)	(1.29)
Same year	-0.0467	-0.0344	-0.0285	-2.251	-1.706	-1.326
	(0.0369)	(0.0361)	(0.0368)	(1.442)	(1.429)	(1.472)
Same entryway	-0.0221	-0.0216	-0.0198	-2.032	-2.006	-1.899
	(0.0261)	(0 .026)	(0.026)	(0.933)	(0.9319)	(0.9124)
Social distance fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Allocator year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Recipient year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Allocator entryway fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Recipient entryway fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	905	905	905	905	905	905

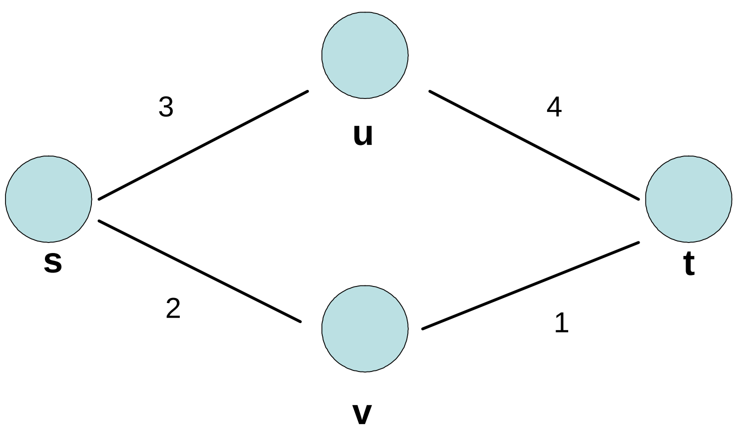
NOTE-- Heteroscedasticity robust standard errors clustered by allocator are reported in parenthesis. Regressions show effect of trust flow measures on selfish behavior in dictator game experiments. See the text for details on the experiment. Dependent variable in columns 1-3 is dummy variable for player 1 (the allocator) keeping entire budget to himself. Dependent variable in columns 4-6 is number of tokens kept by player 1. Allocator sex (recipient sex) equals one if allocator (recipient) is male. Year refers to year in college of allocator or recipient. Entryway refers to staircase or floor of allocator or recipient. Social distance equals number of links in shortest path between allocator and recipient in social network.

FIGURE 1
SOCIAL COLLATERAL IN SIMPLE NETWORKS

A. Three-agent network



B. Four-agent network



NOTE–Figure illustrates the calculation of trust in simple networks. In both panels, agent *s* wishes to borrow an asset from agent *t*. In panel A, the endogenous borrowing limit equals $\min[3, 4] = 3$, which is the value of the weakest link on the path connecting *s* and *t*. In panel B, the borrowing limit is $\min[3, 4] + \min[2, 1] = 4$, the sum of the weakest links on the two paths between *s* and *t*. In general, the borrowing limit is determined by the maximum network flow between *s* and *t*. See the text for details.

FIGURE 2
MODEL TIMELINE

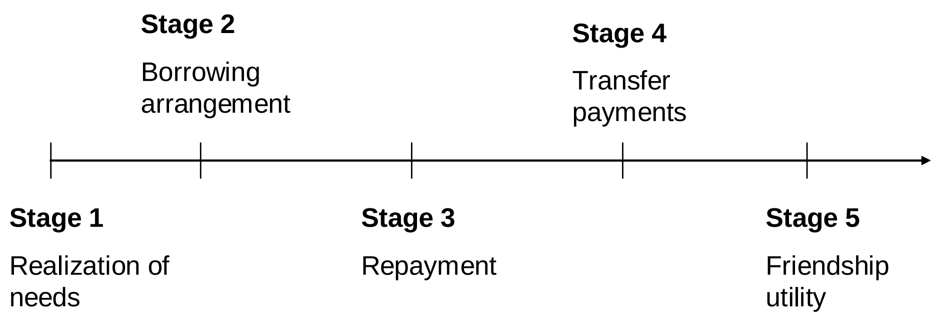
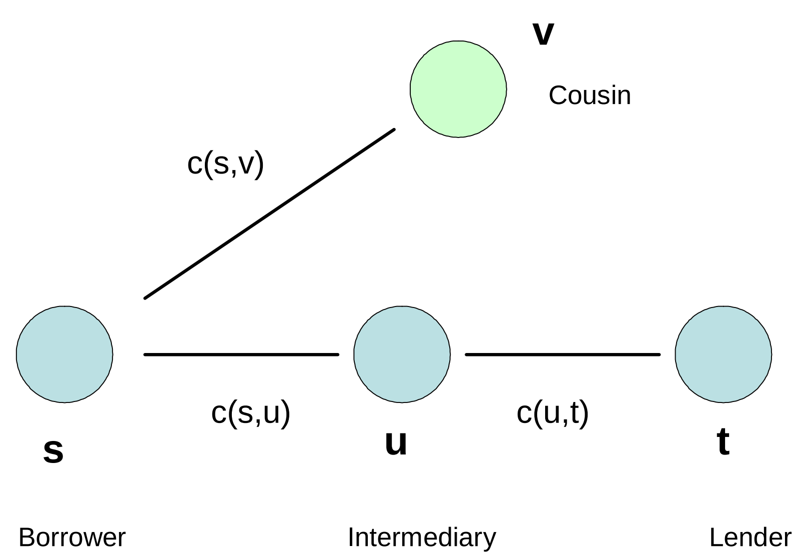
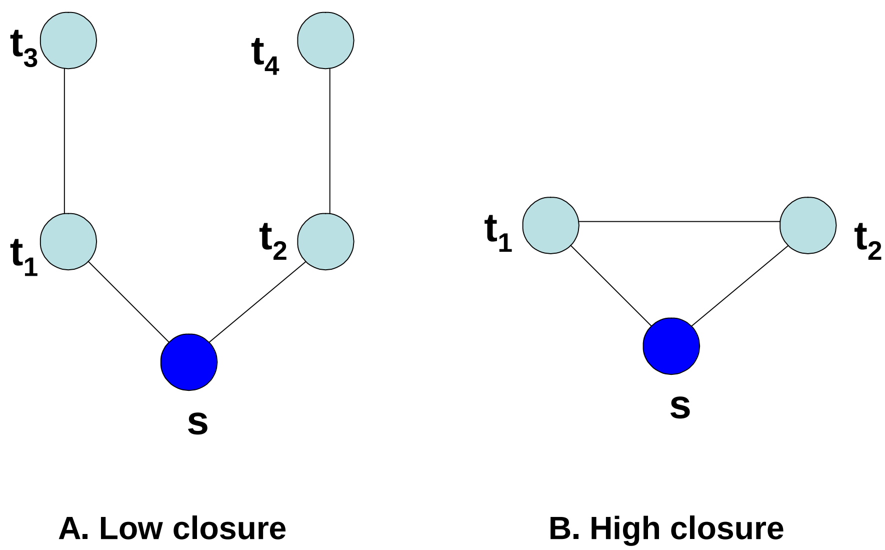


FIGURE 3
BORROWING IN A FOUR-AGENT NETWORK



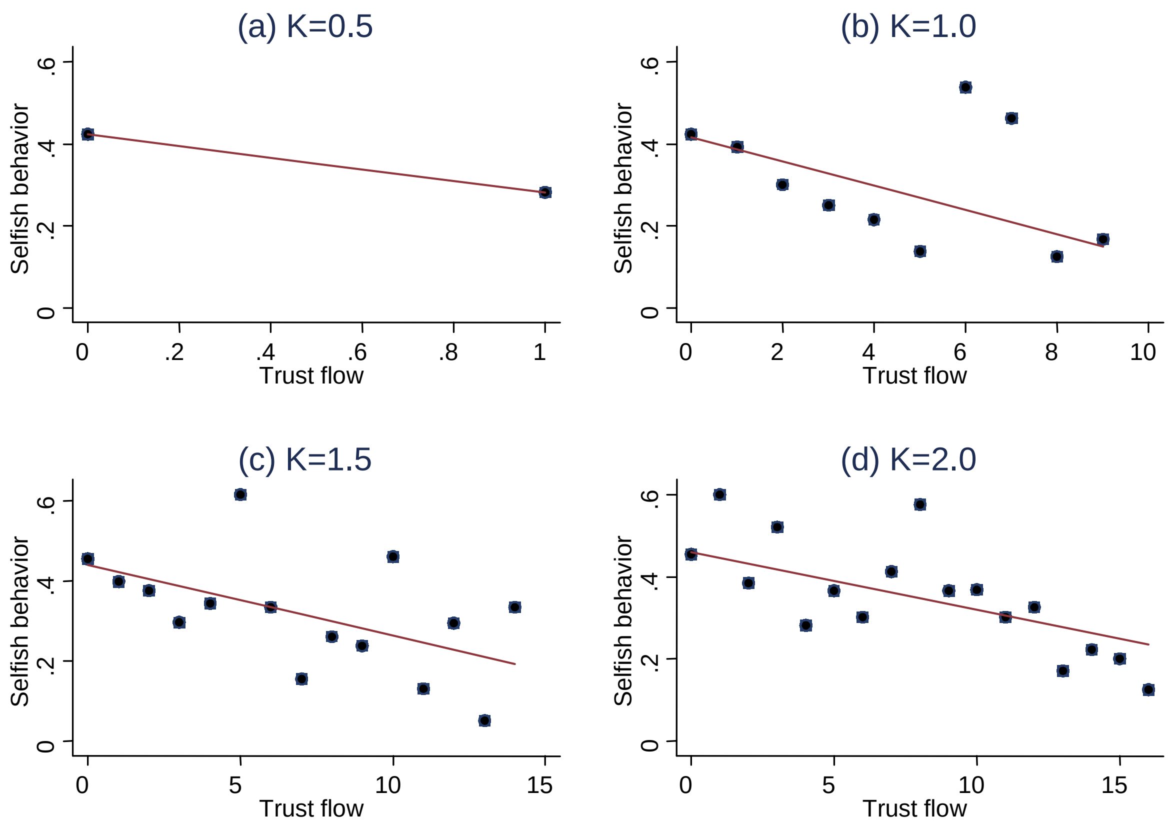
NOTE–Figure illustrates borrowing in networks with intermediaries. The arrangement favored in our paper involves transfers flowing from s through u to t in case of default. In this arrangement the weakest link $\min[c(s,u), c(u,t)]$ is the borrowing limit. An alternative arrangement, where cousin v promises to punish borrower s in case of default, sometimes enforces better outcomes. However, this arrangement is not robust to "side-deals" by groups of agents: the borrower and his cousin can jointly deviate, steal the asset and short-change the lender. As we show in the text, all side-deal proof arrangements satisfy the "weakest link" requirement.

FIGURE 4
NEIGHBORHOODS WITH INCREASING CLOSURE



NOTE–Figure shows network neighborhoods with increasing network closure. The two neighborhoods shown are identical to neighborhoods in Coleman (1988), Figure 1. For $K \geq 1.5$ both neighborhoods generate the same total trust to agent *s*. When technology favors low asset values, the neighborhood in panel A is more attractive because it provides access to more people. When technology favors high asset values, the neighborhood in panel B is more attractive, because closure allows for borrowing high-valued assets.

FIGURE 5
TRUST FLOW AND SELFISH BEHAVIOR
IN DICTATOR GAMES



NOTE–Figure shows relationship between different measures of trust flow developed in the paper and a binary indicator of selfish behavior in dictator game experiments. The four panels correspond to four measures of pairwise trust, for $K = 0.5$, $K = 1.0$, $K = 1.5$ and $K = 2.0$. See the text for the definition of trust measures and details about the experiment. Each panel plots average value of selfish behavior for each possible value of trust flow as well as best fit (OLS) line. The sample used is the same as in Table 2. In panels (b), (c) and (d), the top 1% of observations for trust flow were excluded (less than 10 observations) because less than 3 observations fell in each trust flow bin in this range.