Specific Knowledge and Performance Measurement

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Abstract: I examine optimal incentives, performance measurement and delegation in a model where an agent has specific knowledge (in the sense of Jensen and Meckling) about the consequences of his actions for the principal. Incentive contracts can be based both on measures closely related to the agent’s actions (“inputs”), and a measure closely related to the principal’s payoff (“output”). While input-based pay minimizes the agent’s income risk, only output-based pay encourages the agent to use his knowledge in choosing his actions. To balance these objectives, it is in general optimal to use both performance measures. The results of the analysis provide a common explanation for why central predictions of agency theory, notably predictions implied by the Informativeness Principle, are often not supported by evidence. Moreover, they lead to many novel and testable predictions about properties of incentive contracts and optimal delegation decisions.

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1 Introduction

In modern economies, information is dispersed both across market participants (see Hayek 1945) and within firms. Managers, in particular, usually have unique information about production and market conditions that in practice is too costly to communicate to others in the firm. This type of information, termed “specific knowledge” by Jensen and Meckling (1992), is critical for pricing, investment and many other decisions, and is often the reason why managers are entrusted with decisions in the first place. To deal with the resulting agency problems, it is necessary to provide managers with appropriate incentives.

Little is known, however, about how to design incentives and performance measurement for agents with specific knowledge. Standard models of incentive contracting do not allow for specific knowledge: while an agent’s actions are typically unobservable to his principal, the agent has no private information about what actions he should be pursuing, since the production technology is assumed to be common knowledge.\(^1\) This failure to take into account one of the most important reasons for why principal-agent relationships exist has led prominent empirical researchers to conclude that standard incentive models are too limited for understanding managerial compensation (Murphy, 1999; Lafontaine and Slade, 2000; Lambert, 2001).\(^2\) Attempts to incorporate specific knowledge into incentive models, however, easily run into analytical difficulties.

In this paper I study optimal incentives, performance measurement and delegation in a model in which the agent has specific knowledge, and show that the results have wide-ranging implications. The basic argument captured in the model is very simple: if an agent has specific knowledge about how his actions contribute to the principal’s payoff, then the best way to get him to use his knowledge in choosing his actions is to provide incentives based on the principal’s payoff or another closely correlated measure of “output”. Since output measures are subject to random influences outside the agent’s control, however, output-based pay exposes the agent to income risk, for which the principal has to compensate the agent if he is risk-averse or wealth-constrained. Weighing the benefits

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\(^2\) The importance of localized knowledge is emphasized in papers that study optimal delegation decision; see, for instance, Aghion and Tirole (1997), Dessein (2002), Stein (2002), Marino and Matsusaka (2003), Alonso and Matouschek (2005). But these models typically abstract from incentive contracts, e.g. by assuming that agents are infinitely risk-averse.
and costs of incentive pay then leads to the prediction that the more valuable the agent’s specific knowledge for the principal, the greater are the optimal incentives for the agent.

This basic argument can be expanded to generate predictions about the optimal choice of performance measures. In addition to providing incentives based on output, it is often also possible to base compensation on “input” measures that are closely correlated with an agent’s actions and to a lesser extent subject to noise. For instance, for a salesperson, input measures might include hours worked, number of customers contacted, new accounts created, quality of advice, etc.; and measures of output might be sales or profit (see Section 2 for more examples). The agent’s specific knowledge can then be interpreted as knowledge of how different inputs translate into output. The differences between standard agency models and the scenario considered here, especially with regard to informational assumptions and the available performance measures, are illustrated in Figure 1.

When both input and output performance measures are available, we obtain the following tradeoff, which is formalized in Section 3 for a simplified version of my main model. An incentive contract based on some formula combining input measures minimizes the agent’s income risk, compared with compensation based on a noisy output. But input-based compensation does not give the agent much incentive to use his specific knowledge; instead, his actions are largely determined the compensation plan. Output-based compensation, in turn, gives the agent an incentive to use his knowledge in the most productive way, but also exposes the agent to greater income risk. In the optimal incentive contract, the weights on input- vs. output-based pay are chosen to strike a balance between the dual goals of minimizing income risk and maximizing the utilization of valuable knowledge.

The full model (see Section 4) is a two-task model in which the principal faces technological uncertainty about the productivity of effort for each task. The agent receives private information about the productivities, which he is unable to communicate to the principal. For simplicity, I take the existence of specific knowledge as given; that is, I am not concerned with information acquisition (see Aghion and Tirole 1997) or communication (see Dessein, 2002, and Marino and Matsusaka, 2003).

After receiving his information, the agent chooses his effort levels for both tasks. Depending on the model’s parameters, the agent’s effort can be interpreted literally as multidimensional effort as in Holmström and Milgrom (1991), or alternatively as a decision e.g. between different investment projects like in models of delegation (see Footnote 2).
The agent is risk-neutral but protected by limited liability; it is therefore costly for
the principal to expose the agent to risk. The principal can compensate the agent based
on verifiable information about the agent’s effort (i.e. his input), and on a noisy measure
of output. I refer to the extent of noise in measuring output as environmental uncertainty
(or risk). In contrast to Holmström and Milgrom (1991), I abstract from any difficulties
in measuring the agent’s effort on each task. The main insights of the paper, however,
remain valid even when multitask measurement problems exist.

I obtain a closed-form solution for the optimal compensation contract. When there is
no technological uncertainty, or when the agent has no private information about it, the
principal prefers input-based pay because output-based pay is risky. In contrast, if the
agent’s private information is sufficiently valuable, it is optimal to pay the agent at least
partially for output, even if effort can be measured perfectly.

Giving an agent incentives already implies that he has been delegated authority over
the tasks in question. Although the paper’s emphasis is on the link between specific
knowledge and incentives, the model also lends itself to predictions about the principal’s
optimal delegation decision. Specifically, delegation is optimal if the principal can attain a
higher profit under the optimal incentive contract than if she carried out the tasks by her-
self, without specific knowledge. Delegation and incentives are thus complementary as e.g.
in Holmström and Milgrom (1994). Even when the decision to delegate is inframarginal,
however, changes in the model’s parameters still affect the optimal contract.

The basic idea that the optimal incentive contract depends on the value of the agent’s
information is mirrored in the comparative-statics results, which are derived from one
general principle. Define the social value of knowledge as the difference between the total
surplus attained when an agent with specific knowledge chooses first-best actions, and
the analogous surplus that would result if the agent had no specific knowledge. Then it
can be shown that any parameter change that increases the social value of knowledge also
leads to a shift towards more output-based pay and to a higher profit from delegation.

I go on to argue that the results of the model have many implications, both theoretical
and applied. First, they provide a common explanation for why several predictions of the
standard theory are often not supported by evidence (see Section 5). Prendergast (2002)
already explained, using an argument based on specific knowledge, why the negative
relation between risk and incentives predicted by theory tends to be contradicted by
the data. I show that the same holds for predictions derived from the Informativeness
Principle. For instance, compensation is often based on noisy output measures even when good input measures are available (Murphy 1999, Ittner et al. 2003). Also, changes in performance caused by known events outside the agent’s control are often not filtered out in the way predicted by theory (Bertrand and Mullainathan 2001). Finally, as is well known, relative performance evaluation is rarely used in practice (see e.g. Prendergast, 1999; or Bushman and Smith, 2001). A common explanation can resolve all of these puzzles: when agents have specific knowledge about how to respond to uncertain events, it is optimal to provide output-based incentives even if doing so increases agents’ income risk, which renders the informativeness principle invalid.

Second, the formal results lead to a many empirical predictions (see Section 6), some consistent with the evidence, and some yet to be tested. Specifically, I develop predictions about how optimal incentive contracts and delegation decisions depend on the knowledge gap between principal and agent, job complexity, the value of effort, a firm’s growth options, corporate governance, product market competition, and the rate of change in an industry; and discuss possible empirical measures for the variables of the model.

Two of the predictions follow from theoretical arguments of broader relevance. One is that subjective performance evaluation of an agent by his principal is constrained by the principal’s knowledge about the agent’s job. This observation suggests that subjective evaluations play a different role in practice than envisioned in the previous theoretical literature. Another, unrelated, argument is that if an optimal incentive contract depends on the knowledge gap between principal and agent, it follows very naturally that level and structure of CEO compensation would depend on the quality of corporate governance. In particular, the apparent correlation between weak governance and high CEO pay shown in recent studies does not prove that CEO pay is determined by CEOs’ power to extract rents from shareholders, contrary to what has been claimed.

A third contribution of the paper is to make a step towards synthesizing the theory of incentives and the literature on optimal delegation (see footnote 2). As I argue in Section 4.1, the two-task version of the model helps to eliminate much confusion in the literature over the extent to which “effort exertion” in incentive models and “decision making” in models of delegation are conceptually similar or different.

The formally most closely related model is that of Prendergast (2002), in which an agent has specific knowledge about the profitability of different available projects. When the risk associated with the projects’ payoffs is low, the optimal contract requires the agent
to work on an assigned project and compensates him for his effort. Prendergast shows that as the risk increases, the principal at some point prefers to switch to an “output-based” contract in which the agent is given both the authority and incentives to pick the most profitable project based on his specific knowledge. The present paper contains a similar result among many others, but is very different from Prendergast’s in purpose and scope. Prendergast is concerned solely with the risk-incentives tradeoff. This paper, in contrast, begins with specific knowledge as a ubiquitous feature of principal-agent relationships and explores its consequences for incentive contracting more generally. Most of the puzzles explained and predictions derived here are unrelated to risk.

Other papers in which an agent obtains private information about his productivity before choosing his action include those of Baiman, Larcker and Rajan (1995), Lafontaine and Bhattacharyya (1995), Zabojnik (1996), Baker and Jorgensen (2003), and Shi (2005). Like Prendergast’s, these papers focus on the relation between risk and incentives rather than the role of specific knowledge itself. Also, solving models of this kind is notoriously difficult because the agent’s income risk is a function of his effort, which in turn depends on private information that is learned only after the contract is written. Accordingly, most of the papers mentioned do not obtain analytical results.3

Finally, some papers in the literature on multitask agency problems also examine a tradeoff between input- and output-related pay, or can be interpreted in that way.4 However, it is a tradeoff that arises from an agent’s incentive to “game the system” when performance measures are distorted, which plays no role here. In particular, papers in which the agent has private information about how his effort affects measured performance emphasize the need to suppress the agent’s incentive to use his information (Baker 1992; Baker, Gibbons and Murphy 1994). Here, in contrast, one goal in designing the contract is to encourage the agent’s use of his information.5

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3 Zabojnik (1996) derives an optimal contract analytically but makes a simplifying assumption about the agent’s risk preferences. In independent work, Baker and Jorgensen (2003) obtain some formal comparative-statics results but no closed-form solution for the optimal contract. Their main result is that incentives can be positively related to volatility (what I call technological uncertainty), whereas here they always are. They also show that (as here) optimal compensation is partially based on output even when effort is verifiable, but do not study the determinants of input- vs. output-related pay.


5 Baker (1992) shows that (like here) even with verifiable effort, a forcing contract is generally not
2 Inputs and Outputs: Applications

The novelty of the distinction between “inputs” and “output” emphasized here lies not in the terms themselves (which have been used before), but in the idea that the agent has specific knowledge about how inputs translate into output. The distinction can be applied to any performance measures that can be meaningfully ranked along an input-output continuum, irrespective of how close the measures are to the ends of the spectrum (effort and actual output). For any two performance measures for a given job, one can label the measure more closely related to the agent’s actions as an “input”, and the measure more closely related to the principal’s payoff as an “output”. Whether a performance measure is to be classified as an input or output depends on what other measure it is being compared with. The same performance measure may be called an input for one job but an output for another. Some examples may be helpful:

Two of the most often studied performance measures for CEOs are stock prices and accounting (e.g. earnings) measures. Stock price is usually the best available measure of firm value, but is also influenced by many factors outside a CEO’s control. Accounting and non-financial (e.g. operational) measures, on the other hand, are less closely related to firm value but are more under the CEO’s control because “managers understand and can ‘see’ how their day-to-day actions affect year-end profitability” (Murphy 1999, p. 2506). Accordingly, one can think of stock price as output, and accounting and non-financial measures as inputs.

If CEOs have specific knowledge of how they contribute to firm value, then shareholders face a tradeoff between undistorted but risky stock-based compensation, and compensation based on other measures that is less risky but also gives CEOs less incentives to use their knowledge to maximize firm value.

Similar arguments apply to the distinction between Economic Value Added (EVA) and scores obtained from a “Balanced Scorecard”, both of which have been advocated in the accounting literature and among practitioners as performance measures for executives. Here, EVA, as a highly aggregated accounting measure, is an output measure compared optimal if the agent has private information about how his effort affects measured performance. But without distortions a fully output-based contract is optimal since in his model the agent is risk-neutral.

6 See also Sloan (1993), who argues that firms’ use of accounting measures in addition to stock price can be understood as a way to shield CEOs from market risk.

7 Clearly, not all non-financial measures are inputs. For example, market share would hardly be considered an “input” relative to (output) measures such as share price or earnings.
with the wide array of financial and non-financial performance measures provided by a balanced scorecard. Again, specific knowledge plays a key role in the distinction: scorecard measures may be more accurate, but firms do not know how they translate into firm value, and therefore end up weighting them in an arbitrary manner. As a result, “tying rewards only to the scorecard metrics exacerbates the problem employees have in making multiple tradeoffs without having a definition of what better is” (Stern Stewart, 1999). By contrast, basing compensation on EVA exposes managers to risk but gives them much better incentives to use their knowledge.

For division- or lower managers, “output” is often best captured by (accounting) measures of division profit, whereas “inputs” may include a variety of operational measures.\(^8\) Alternatively, when managers are compensated based on the performance of their own unit as well as that of a larger group (e.g. the entire firm), one can think of the former as an input and the latter as an output (see Section 6, point 6).

Similar to retail chains face a choice between compensating the managers of company-owned outlets based on sales figures or instead operational measures (such as “quality, service, cleanliness” in the classic McDonalds case, see Sasser and Pettway, 1974). Salespeople can be compensated based on the sales or profit they generated (output), but also on very detailed input measures such as new accounts created, hours worked, number of items sold in certain product categories, etc.\(^9\)

### 3 A Simple One-task Model

This section develops the main insights of the paper in a simple model with one task and only few parameters. In the next section, I study a more elaborate two-task version of the model, which leads to additional insights. Consider a model in which a principal hires an agent to produce an output by exerting effort:

\(^8\) Ittner and Larcker (1998) document an increase in the use of non-financial in addition to the more traditional financial performance measures over time. Ittner, Larcker and Rajan (1997) study the determinants of the use of financial and non-financial measures, testing hypotheses derived from the informativeness principle.

\(^9\) Hauser, Simester and Wernerfelt (1994) have argued that measures of customer satisfaction may be better measures of long-term profitability from a salesperson’s efforts than short-term sales. In line with a main point of this paper, they argue that basing compensation on customer satisfaction may be desirable especially when the agent knows better than the principal how his actions contribute to long-run profits.
Production: Output, denoted $Y$, is stochastic and can be either 0 or 1. The probability that $Y = 1$ is given by $\min\{a\theta, 1\}$, where $a$ is the agent’s effort exerted on the two tasks, $\theta$ is the productivity of effort.

Technological uncertainty: The productivity $\theta$ is either high, $1 + t$, or low, $1 - t$, with equal probability. The parameter $t \in [0, 1]$ measures the degree of technological uncertainty. (Many of the model’s parameters are normalized between 0 and 1 or $-1$ and 1, which significantly simplifies the subsequent calculations.)

Information about $\theta$: The principal knows the distribution but not the realization of $\theta$. The agent receives a private signal $\tilde{\theta} \in \{1 + t, 1 - t\}$ about $\theta$. The probability that $\tilde{\theta} = \theta$ is given by $(1 + k)/2$, where $k \in [0, 1]$ captures the quality of the agent’s knowledge. The agent cannot communicate $\tilde{\theta}$ to the principal.

The productivity $\theta$, and the agent’s specific knowledge about it, should be interpreted very broadly; it may represent information about the firm’s production technology, market conditions, or optimal responses to changes in those conditions. All that matters is that the agent has specific knowledge relevant to his (first-best) choice of $a$, which is what the multiplicative term $a\theta$ is meant to capture.

Agent’s utility: The agent is risk-neutral, but protected by limited liability, which is a combination of assumptions economically very similar to risk aversion. The agent’s utility is $w - a^2/2$, where $w$ is his total compensation (described below) and $a^2/2$ is the disutility of effort.

Performance measurement: As discussed in Section 2, the distinction between inputs and output can be interpreted very broadly. But since the role of measurement errors for the design of optimal contracts is well known (see Prendergast, 1999, or Lafontaine and Slade, 2000), there is no loss of generality in assuming in the model that one performance measure is effort itself, and the other a noisy measure of output. This simplification also helps to highlight a central point of this paper: even if effort can be perfectly measured, basing compensation on risky output measures is generally desirable when the agent has specific knowledge.

Thus, I assume that the agent’s effort $a$ can be measured without noise and is contractible. Output, however, can be measured only imperfectly by a contractible variable $y$ that takes the values 0 or 1. The probability that $y = Y$ is given by $(2 - e)/2$, where $e \in [0, 1]$ is a measure of environmental uncertainty or risk: if $e = 0$, $y$ measures $Y$ per-
fectly, whereas if $e = 1$, $y$ is entirely uninformative. Unless mentioned otherwise, I assume that $e > 0$, since for $e = 0$ there is no reason to use $a$ as a performance measure and the results of the model are trivial. For $\theta a \in [0, 1]$, the expected value of $y$ conditional on $\theta$ and $a$ is given by

$$E[y(\theta, a)] = \frac{e}{2} + (1 - e)\theta a. \quad (1)$$

Note from (1) that while environmental risk does not affect the true productivity of the agent’s effort, it does reduce the responsiveness of measured performance to the agent’s effort. This is a standard feature of any moral-hazard model with only two possible outcomes (see e.g. Laffont and Martimort 2001, Chapter 4.3), but stands in contrast to e.g. the model of Holmström and Milgrom (1987, 1991).\(^{10}\)

**Compensation:** I confine attention to contracts that are linear in both $a$ and $y$; that is, the agent’s total compensation is given by

$$w = \alpha + \beta a + \gamma y. \quad (2)$$

The restriction to linear contracts is standard, and is motivated by their use in practice. Also, keep in mind that $a$ used as performance measure stands as a metaphor for noisy signals of effort, which is why I assume that the principal pays a piece rate for effort instead of using e.g. a forcing contract (it will become clear that a forcing contract would not be optimal anyway).

Limited liability means that the agent’s compensation must be non-negative for any effort level and any realized output (although realized utility may be negative). Since both $a$ and $y$ can be zero, this means that the salary $\alpha$ must be nonnegative. As is standard in models with limited liability, I assume that the agent’s participation constraint is not binding; therefore, we can without loss of generality set $\alpha = 0$.

**Timing:**

1. The principal offers a contract $(\beta, \gamma)$; the agent accepts or rejects.

2. The productivity $\theta$ is realized.

\(^{10}\) Another source of uncertainty in addition to technological uncertainty and environmental risk is of course the randomness of $Y$ itself. This randomness, however, is irrelevant for the results because both principal and agent are risk-neutral and therefore only care about the expected value of $Y$. The randomness of $Y$ is merely a convenient way to map continuous actions into a set of two outcomes.
3. The agent receives a signal \( \tilde{\theta} \) about \( \theta \).

4. The agent chooses effort \( a \).

5. The output \( Y \) and the measured performance \( y \) are realized, and the agent is compensated accordingly.

To keep the calculations simple and avoid case distinctions, I restrict the analysis to parameters that lead to interior equilibrium solutions for all endogenous variables. Necessary and sufficient conditions are

\[(a1) \quad 2k^2t^2 > \frac{e}{1-e}, \quad \text{and} \]
\[(a2) \quad 2(1-e)kt[2 - (1+t)(1+kt)] + e(1+t) > 0 \]

Condition (a1) states a lower bound to the quality of the agent’s knowledge \( k \), and guarantees an interior solution for \( \gamma \). Condition (a2) guarantees that \( \theta a \) does not exceed 1.\(^{11}\)

The analysis of the game is very simple. In fourth stage of the game, the agent chooses his effort based on his beliefs about \( \theta \), denoted \( \hat{\theta} \), to maximize

\[ \beta a + \gamma E(y|\hat{\theta},a) - \frac{a^2}{2} = \beta a + \gamma \left( \frac{e}{2} + (1-e)\hat{\theta}a \right) - \frac{a^2}{2}, \quad (3) \]

the solution to which is

\[ a = \beta + (1-e)\gamma \hat{\theta}. \quad (4) \]

To compute various expected values, a slightly different notation is convenient: The productivity \( \theta \) can be written as \( 1 + \tau t \), where \( \tau = \pm 1 \) with equal probability. Likewise, the agent’s signal \( \tilde{\theta} \) can be written as \( 1 + st \), where \( s = \pm 1 \) and \( \Pr(s = \tau) = (1+k)/2 \). It is easy to establish that the agent’s belief \( \hat{\theta} \) conditional on \( s \) is then given by \( \hat{\theta} = 1 + kts \).

All that remains to do is to plug (2), (1) and (4) into the principal’s profit \( Y - w \) and take expectations over the joint realizations of \( \tau \) and \( s \). Maximizing the resulting expected profit over \( \beta \) and \( \gamma \) leads to the following result:

**Proposition 1** Under conditions (a1) and (a2), the optimal contract parameters are given by

\[ \beta^* = \frac{e}{4(1-e)k^2t^2} \quad \text{and} \quad \gamma^* = \frac{1}{2(1-e)} - \frac{e}{4(1-e)^2k^2t^2}, \quad (5) \]

where \( \gamma^* \) is increasing and \( \beta^* \) is decreasing in \( k \).

\(^{11}\) The existence of parameter sets that satisfy both conditions can be seen by example; choose e.g. \( k = .8, t = .6 \) and \( e = .2 \).
Proof: see the Appendix.

At one extreme, when pay is only input-based ($\beta > 0$, $\gamma = 0$), then the agent’s effort depends only on $\beta$ and not on his private information $\tilde{\theta}$; cf. (4). Thus, with input-based pay, the agent has no incentive to use his specific knowledge. This is still an incentive contract and not a forcing contract; nevertheless, the agent’s equilibrium effort is fully determined, very much like in standard models of incentive contracting.

At the other extreme, purely output-based pay ($\beta = 0$, $\gamma > 0$) is costly for the principal whenever measuring output is subject to errors ($e > 0$): the more measured performance is affected by noise, the less it is affected by effort, cf. (1). To induce a certain level of effort, the principal therefore has to pay a higher reward for a good outcome. Since the reward for a bad outcome is already bounded from below at zero, greater risk is costly for the principal.\footnote{This result is familiar when the agent is risk-averse, but also holds when the agent is risk-neutral but protected by limited liability (on the following, see also Laffont and Martimort, 2001, Chapter 4). While in the model of Holmström-Milgrom (1987) risk affects the agent’s participation constraint but not his incentive constraint, in general (and especially in models with only two realizations of performance) it affects both constraints. With a risk-neutral agent and limited liability, risk affects only the incentive constraint.}

Thus, the key tradeoff in this model is that input-based pay is riskless but fails to make use of the agent’s information, whereas output-based pay encourages the optimal use of information but is risky and hence costly for the principal. When technological uncertainty and the agent’s specific knowledge are sufficiently important, the optimal incentive contract is partly input- and partly output-based.

Proposition 1 also states the most important comparative-statics result of the model: the better the agent’s knowledge, the more his compensation will be based on output; the intuition is straightforward. This result also holds when output is the only performance measure available, i.e. when input cannot be measured:

**Proposition 2** If $a$ is not verifiable and hence $\beta$ is restricted to zero, then the optimal output piece rate is given by

$$\gamma^* = \frac{1}{2(1-e)} - \frac{e}{4(1-e)^2(1 + k^2t^2)},$$

which is increasing in $k$.

Proof: see the Appendix.
While Propositions 1 and 2 are intuitive, their significance is to establish a link between specific knowledge and the properties of optimal incentive contracts. I leave a discussion of how the optimal contract depends on the uncertainty parameters $e$ and $t$ to Sections 4.4 and 5.1 in the context of the full model.

4 The Full Model With Two Tasks

I now extend the basic model to two tasks. In the two-task model, the agent must be induced not only to exert a certain level of effort, but also to allocate his effort in an efficient manner. I introduce some new notation and additional parameters; otherwise, everything is the same as in the one-task model.

4.1 Model

Production: The output $Y$ is again either 0 or 1; the probability that $Y = 1$ is given by $\min\{a_1\theta_1 + a_2\theta_2, 1\}$, where $\mathbf{a} = (a_1, a_2)$ is the agent’s effort exerted on the two tasks, and $\mathbf{\theta} = (\theta_1, \theta_2)$ is a vector of productivities.

Technological uncertainty: The productivity $\theta_i$ of each task $i$ is given by $\theta_i = \bar{\theta}(1 - t\tau_i)$, which differs from the basic model only in that $\theta_i$ is scaled by the parameter $\bar{\theta}$. The $\tau_i$, and thus the $\theta_i$, may be correlated; the probability that $\tau_1 = \tau_2$ is given by $(1 + \rho)/2$ for $\rho \in [-1, 1]$. When $\rho = 1$, the model in effect collapses to a one-task model. When $\rho = -1$, the total productivity of effort across tasks is always constant but is concentrated on one task or the other.

Information about $\theta$: The same assumptions as before hold. The principal knows the expected productivity $\bar{\theta}$, but not the realization of $\mathbf{\theta}$. The agent receives a private signal $\mathbf{s} = (s_1, s_2)$ about $\mathbf{\tau}$; the probability that $s_i = \tau_i$ is given by $(1 + k)/2$. The $s_i|\tau_i$ are independent, and so $s_1$ and $s_2$ are correlated only indirectly through the correlation between $\tau_1$ and $\tau_2$.

Agent’s utility: The agent’s utility is $w - d(\mathbf{a})$; the disutility $d(\mathbf{a})$ is given by

$$d(\mathbf{a}) = \frac{d}{1 + \phi}(a_1^2 + a_2^2 + 2\phi a_1 a_2),$$

for $\phi \in (-1, 1]$. If $\phi = 1$, the tasks are perfect substitutes for the agent in the sense that $d(\mathbf{a})$ reduces to $d(a_1 + a_2)^2/2$. If $a_1 = a_2 = a$, then $d(\mathbf{a})$ reduces to $2da^2$. Thus, scaling the
disutility by $1/(1 + \phi)$ ensures that changes in $\phi$ affect the interaction $\partial^2 d(a)/(\partial a_1 \partial a_2)$ but not the level of disutility for equal levels of effort on each task.

**Performance measurement and compensation:** Compensation can be based on both $a$ and a noisy measure of output $y$, which relates to true output $Y$ in the same way as in the basic model. Hence, for $\theta_1 a_1 + \theta_2 a_2 \in [0, 1]$, the expected value of $y$ conditional on $\theta$ and $a$ is given by

$$E[y(\theta, a)] = \frac{e}{2} + (1 - e)(\theta_1 a_1 + \theta_2 a_2).$$

Contrasts are restricted to be linear; the agent’s total compensation is given by

$$w = \alpha + \beta(a_1 + a_2) + \gamma y.$$ 

Again, we can set $\alpha = 0$. The use of the same piece rate for $a_1$ and $a_2$ is not a restriction but follows as a result from the symmetry of the model. The timing of the game is the same as before.

Aside from introducing the new and interpretable parameters $\tilde{\theta}$, $\rho$, $d$, and $\phi$, the two-task version of the model helps to clarify the relation between “effort” in models of incentive contracting and “decision making” in models of delegation. Critics sometimes question the usefulness of standard hidden-action models, with their focus on effort provision, for the problem of how to induce managers to “make the right decisions”, for instance about the choice of investment projects (Holmström 1992, Murphy 1999, Lambert 2001). Some even consider effort and decision-making to be entirely different things, where specific knowledge plays a role for the latter but not the former.\(^{13}\)

This distinction already suggests that the real problem with standard incentive models lies not in the notion of effort, but instead a failure to account for the role of specific knowledge. Once it is incorporated into the model, the distinction between effort and decision-making reduces to a distinction between “how much” and “what”, while the principles of optimal compensation are the same for both: when agents are risk-averse, they should be shielded from risk they cannot respond to, irrespective of whether it pertains to measures of their effort or of their project choices. And their incentives should be aligned with the principal’s objectives irrespective of whether their choice is about

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\(^{13}\) See, for instance, Athey and Roberts (2001), who argue that “motivating effort is done best by rewarding agents on precise measures of their effort... At the same time, ... getting the right investment choices may require that the decision-maker’s rewards be tied to total value created” (p. 200).
effort or projects. Thus, the choice of investment projects is meaningfully different from
“effort” only when there are implications beyond expected profit (such as risk) to consider.

The model introduced here fits both interpretations. If \( \rho = 1 \), we have the pure
“effort” case in the sense that only the level but not allocation of effort matters. The
opposite case, \( \rho = -1 \), can be interpreted as a situation where the agent must choose
between two projects, and exactly one is profitable. With perfect knowledge \( (k = 1) \), the
agent then chooses one of two (symmetric) effort allocations, which can be interpreted as
choosing one of the projects. With less than perfect knowledge, the agent will generally
hedge his bets and invest in both projects (depending on the information he receives),
but that does not affect the interpretation.\(^{14} \)

### 4.2 Optimal Contract

Define

\[
\eta = \frac{2(1 - \phi)\rho(1 - k^2\rho) + (1 - \rho)^2(1 + k^2\phi\rho)}{1 - k^4\rho^2}.
\]

Then the following conditions are necessary and sufficient for the existence of an interior
solution for the principal’s optimal contract:

\begin{align*}
(A1) & \quad 2(1 - e)k^2t^2\eta \bar{\theta}^2 > (1 - \phi)de \\
(A2) & \quad (1 - \phi^2)(1 - \rho)de + 2(1 - e)\eta kt\bar{\theta}^2[(1 - \phi)(1 - k^2\rho) - (1 + \phi)(1 - \rho)kt] > 0 \\
(A3) & \quad 2(1 - e)(1 + k^2\rho)\eta kt[2d - (1 + t)\bar{\theta}^2] - (1 + t)(1 + \rho)[2(1 - e)\eta k^2t^2\bar{\theta}^2 - de(1 - \phi)] > 0
\end{align*}

Assumption (A1) ensures that the optimal \( \gamma \) in the contract is positive. Just as Condition
(a1) in the previous section, it states a lower bound for \( k \) relative to \( e \). Assumption (A2)
ensures that for any realization of \( s \), the agent chooses positive levels of effort for each
task. A necessary condition is that \( \phi < 1 \); at the end of this subsection I explain why this
is a technical assumption that has no economic significance for the results.\(^{15} \) Assumption

\(^{14} \) Note, though, that if \( k < 1 \), then the agent’s total effort is not constant even if \( \rho = -1 \), since the
agent’s effort depends on his signals \( (s_1, s_2) \) (which can be both good or both bad) and not just \( \theta \). Thus,
even if \( \rho = -1 \), there is a “vertical dimension” to the agent’s choice of \( a \) in addition to the “horizontal”
allocation between tasks 1 and 2. See also the discussion at the end of Section 4.2.

\(^{15} \) Intuitively, if the tasks’ productivities differ considerably, or the agent’s knowledge is very good, or
the agent cares little about how he allocates his effort across the two tasks, then he will devote all effort
to task \( i \) and none to task \( j \neq i \) if \( s_i > s_j \).
(A3), finally, ensures that $a_1 \theta_1 + a_2 \theta_2$ never exceeds 1.\footnote{That parameters satisfying (A1)-(A3) exist can be proven by example: take $\bar{\theta} = 1$, $e = .1$, $t = k = .5$, $d = 1$, $\rho = 0$, and $\phi = .5$, in which case $\eta = 1$.}

Deriving the optimal contract is as straightforward as for the one-task version, but some computations become more cumbersome:

**Proposition 3** Under (A1)-(A3), the optimal contract parameters are given by

$$
\beta^* = \frac{(1 - \phi)de}{4(1 - e)k^2t^2\eta \theta} \quad \text{and} \quad \gamma^* = \frac{1}{2(1 - e)} - \frac{(1 - \phi)de}{4(1 - e)^2k^2t^2\eta \theta^2}.
$$

\footnote{That parameters satisfying (A1)-(A3) exist can be proven by example: take $\bar{\theta} = 1$, $e = .1$, $t = k = .5$, $d = 1$, $\rho = 0$, and $\phi = .5$, in which case $\eta = 1$.}

**Proof:** see the Appendix.

The intuition for this result is the same as for Proposition 1: the levels of $\beta$ and $\gamma$ jointly determine the level of the agents’ effort, while the relative weights on $\beta$ and $\gamma$ balance the dual objectives of getting the agent to use his knowledge (higher weight on $\gamma$) and minimizing his rent due to limited liability (higher weight on $\beta$).

I discuss in Section 4.4 how the optimal contract depends on the parameters of the model. Like in Section 3, it is straightforward to derive an optimal output piece rate $\gamma^*$ when output is the only available performance measure; the results are omitted.

A potential misunderstanding about Proposition 3 is the idea that the optimality of output-based pay may depend on the agent having personal preferences over the tasks, i.e. on $\phi$ being less than 1. For if the agent were indifferent between his tasks ($\phi = 1$), so the argument goes, an input-based contract with some arbitrarily small output bonus would suffice to get the agent to act in the principal’s interest by allocating his effort according to his private information.

Although, as mentioned, (A2) does require that $\phi < 1$, the above argument is incorrect. It would be valid if the agent could somehow exert a fixed amount of total effort (at a cost) and then freely allocate it between the two tasks. Here, however, the agent’s disutility function is convex in effort (a more natural assumption), and his (first- and second-best) total effort is not constant but varies with $\tilde{\theta}$, except if $\rho = -1$ and $k = 1$. Thus, the purpose of output-based pay is to induce the agent to exert a high level of effort if and only if he has received a signal that his effort is productive. In fact, the optimal $\gamma$ is increasing in $\phi$, see Section 4.4.

Although (A2) ceases to hold as $\phi$ approaches 1, the analysis of this case is conceptually straightforward and leads to a closed-form solution for the optimal contract; the resulting
expressions are more complicated than those derived below. The comparative-statics analysis is more complicated, too, but leads to results very similar to those presented below. Hence, while the case $\phi = 1$ is ruled out in this paper to ensure an interior solution, *economically* the results do not depend on the assumption that $\phi < 1$.

4.3 The Principal’s Delegation Decision

As I argued earlier, the dispersion of important knowledge within organizations is one of the main reasons why principal-agent relationships exist in the first place. It is therefore natural to ask under what circumstances the principal would indeed want to delegate the tasks in our model, i.e. the choice of actions $a$, to the agent, the alternative being for the principal to perform these task by herself (equivalently, she might employ another agent on a forcing contract at zero monitoring cost).

Unfortunately, we cannot meaningfully endogenize this decision. The main tradeoff of interest here is that by not using an agent, the principal can save on agency costs but forgoes the utilization of the agent’s knowledge. In addition, however, delegation decisions are driven not only by the agent’s knowledge, but by differences between agent and principal in productivity or the value of time as well. For example, the agent might be generally more skilled in the tasks than the principal, corresponding to a larger $\theta$. Or the principal’s time might be more valuable than the agent’s, corresponding to a larger $d$ for the principal. A direct comparison of the two scenarios is therefore meaningless unless we know how the principal’s and the agent’s productivity and value of time are related.

What we can do is make predictions about how changes in the model’s parameters affect the principal’s profit under delegation, and thus the likelihood of delegation. Denote by $E(\pi)$ the principal’s expected profit as stated in (30) in the proof of Proposition 3, with $\beta = \beta^*$ and $\gamma = \gamma^*$. Assuming that the model’s parameters can vary independently of the corresponding parameters for the principal, any parameter change leading to an increase in $E(\pi)$ then shifts the principal’s decision towards delegation.

Thus, changes in the parameters of the model affect $E(\pi)$ and hence may affect the principal’s decision whether to delegate the tasks to an agent at all. Whenever $E(\pi)$ exceeds the principal’s non-delegation profit, however, parameter changes only affect the optimal incentive contract. In other words, while delegating tasks to an agent requires

\[17\] It is available in a web-Appendix at http://www.simon.rochester.edu/fac/raith/papers/Skapm5_AppB.pdf
providing him with incentives, different job environments may give rise to different incentive contracts without any difference in the assignment of tasks. This situation is likely given for many managerial positions endowed with extensive decision rights.

4.4 The Social Value of Knowledge and Comparative Statics

I now show that how most parameters of the model affect the optimal contract and delegation decision depends on how they affect what I define as the social value of knowledge.\(^{18}\) Subsequently, I discuss results for individual parameters.

Since the agent’s wage \(w\) is a pure transfer, the total surplus between principal and agent is given by the output \(Y\) minus the agent’s disutility, which for given expectations \(\hat{\theta}(s)\) is

\[
 a_1\hat{\theta}_1 + a_2\hat{\theta}_2 - \frac{d}{1 + \phi} (a_1^2 + a_2^2 + 2\phi a_1 a_2). \tag{10}
\]

Maximization of (10) with respect to \(a_1\) and \(a_2\) leads to the first-best effort levels:

\[
a_{1}^{FB}(\hat{\theta}) = \frac{\hat{\theta}_1 - \phi \hat{\theta}_2}{2d(1 - \phi)} \quad \text{and} \quad a_{2}^{FB}(\hat{\theta}) = \frac{\hat{\theta}_2 - \phi \hat{\theta}_1}{2d(1 - \phi)}. \tag{11}
\]

By substituting (25) and (11) into (10) and taking expectations over \(\theta\) and \(s\), one obtains the expected total surplus if an agent with knowledge \(k\) uses his information optimally according to (11). Denote this surplus by \(W(k)\); its expression is given in the proof of the next result. Then we can define the social value of knowledge as the difference between \(W(k)\) and the corresponding surplus if the agent had no specific knowledge, i.e. \(W(0)\).

**Proposition 4** The social value of knowledge \(v\), defined as \(W(k) - W(0)\), is given by

\[
v = \frac{\eta k^2 t^2 \hat{\theta}_2}{2d(1 - \phi)}. \tag{12}
\]

It is increasing in \(\hat{\theta}\), \(k\) and \(t\), decreasing in \(d\) and increasing in \(\phi\).\(^{19}\) Also, \(v\) is decreasing in \(\rho\) if and only if

\[
k^2 \phi(1 - \rho^2)(1 - k^2 \rho^2) + 2(1 - k^2) [\phi(1 - k^2 \rho^2) - \rho(1 - k^2)] \geq 0, \tag{13}
\]

There exists \(\tilde{\rho} \in (0, 1)\) such that (13) holds if and only if \(\rho \leq \tilde{\rho}\). A sufficient condition for (13) to hold is that \(a_{1}^{*}(\theta)\) is larger when the agent obtains a bad signal on task 2 than when he obtains a good signal.

\(^{18}\) I would like to thank Meg Meyer for suggesting this generalization.

\(^{19}\) Recall from (8) that \(\eta\) depends on \(k\), \(\rho\) and \(\phi\).
Proof: see the Appendix.

I will explain the intuition for the different parts of Proposition 4 below after Proposition 5. Both the optimal contract and the principal’s delegation decision are closely connected to the social value of knowledge. From (9) and (12) we immediately obtain

\[ \beta^* = \frac{e \hat{\theta}}{8(1 - e)v} \quad \text{and} \quad \gamma^* = \frac{1}{2(1 - e)} - \frac{e}{8(1 - e)^2v}. \]  

Equation (14) shows that most parameters changes affect the optimal contract through a change in the value of information \( v \). Notice also that changes in \( v \) lead to changes in \( \beta^* \) and \( \gamma^* \) in opposite directions. This allows us to make unambiguous predictions about the optimal relative weights placed on input and output performance measures for all parameters except \( e \).

The principal’s expected delegation profit (30), too, can be restated as a function of the value of knowledge:

\[ E(\pi) = \frac{1}{d} [\beta^* + (1 - e)\gamma^*] \{\hat{\theta} - [\beta^* + (1 - e)\gamma^*]\} - \frac{e}{2} \gamma^* + 2[1 - (1 - e)\gamma^*] \gamma^* v. \]  

We then obtain:

**Proposition 5**

(a) \( \gamma^* \) is increasing in \( k, \tilde{\theta}, t \) and \( \phi \). It is decreasing in \( d \), decreasing in \( e \) if \( \partial E(y)/\partial e \geq 0 \), and decreasing in \( \rho \) if and only if (13) holds.

(b) \( \beta^* \) is decreasing in \( k, \tilde{\theta}, t \) and \( \phi \). It is increasing in \( d \) and \( e \), and increasing in \( \rho \) if and only if (13) holds.

(c) The principal’s delegation profit \( E(\pi) \) is increasing in \( k, \tilde{\theta}, t \) and \( \phi \). It is decreasing in \( d \) and \( e \), and decreasing in \( \rho \) if and only if (13) holds.

Proof: see the Appendix.

The results for \( \gamma^* \) are exactly the same if only output-based compensation is feasible; the details are omitted. Except for the results for \( e \), Proposition 5 is based on Proposition 4 and equations (14) and (15). This means that there is a common intuition for almost all results: Any change in a parameter that increases the social value of the agent’s information leads to a shift towards more output-based pay, and makes delegation more likely. Conversely, any parameter change that leads to (say) an increase in output-based pay can be explained by an increase in the value of the agent’s information as a result of the parameter change. This is straightforward in the case of the agent’s knowledge \( k \) but also holds for the other parameters.
For instance, since $\bar{\theta}$ scales the productivity of effort, a higher value of $\bar{\theta}$ is associated with a higher value of the agent’s knowledge about how to allocate his effort. It follows that the relative weight on output-based pay and the delegation profit are increasing in $\bar{\theta}$. A higher $t$ implies a greater variance of the tasks’ productivities, which makes it more valuable for the principal to rely on the agent’s knowledge about them, similar to Prendergast’s (2002) main result. As for $\rho$, if the productivities of the tasks are not already highly correlated, then the lower $\rho$, the higher the value of the agent’s information, since it is more likely that the agent should focus his effort on one task rather than both. The parameter $\phi$, in turn, measures how substitutable the two tasks are from the agent’s point of view. The higher $\phi$, the less strongly the agent cares about what tasks he works on, and hence the more the agent allocates his effort according to his private information rather than his personal preferences. It is then optimal to put a larger weight on output-based compensation.

The results for environmental risk ($e$) are not based on Proposition 4 but are familiar from standard agency models (see also Footnote 12): the greater the noise in measuring output, the lower the optimal weight on output in the incentive contract, and the lower the principal’s delegation profit. The optimal contract depends on $e$ although the social value of information does not: risk affects measured output $y$ but not true output $Y$; consequently, first-best effort and hence $v$ do not depend on $e$. To understand the qualifying condition for $\gamma^*$ in part (c), notice that a change in $e$ affects the variance of $y$, but therefore also (as in any model with only two possible outcomes) its expected value. Clearly, other things equal, the principal would want to decrease $\gamma$ if $E(y)$ increases, and vice versa. The question then is what happens when $\partial E(y)/\partial e = 0$. Part (a) states that in this case, an increase in $e$ unambiguously leads to a decrease in $\gamma$. This effect is reinforced for higher values of $e$, where $E(y)$ is increasing in $e$, whereas for smaller values it may be outweighed by the upward adjustment of $e$ because of a decrease in $E(y)$.

Part (c) of Proposition 5 states the intuitive result that delegation is more profitable the larger the agent’s information advantage as measured by $k$. In contrast, other models of delegation usually assume that the agent has specific knowledge, but do not study variations in the quality of knowledge (see e.g. Dessein 2002, Prendergast 2002). More importantly, in my model incentive contracting plays a central role, and the delegation decision itself depends on the delegation profits attainable under the optimal contract. Part (c) confirms a central conjecture of Jensen and Meckling (1992) that if local knowledge is
important and costly to transfer (i.e. specific), firms will assign decision rights to agents who have the relevant knowledge to make those decisions. For recent evidence supporting this prediction, see e.g. Christie, Joye and Watts (2003), Colombo and Delmastro (2004), and Abernethy et al. (2004).

Section 5.1 below draws on the results of Proposition 5 for $e$ and $t$. A much more detailed discussion of the implications of Proposition 5 appears in Section 6.

5 A Common Explanation for Empirical Puzzles

In his survey on incentives in firms, Prendergast (1999) concluded that while there is little doubt that people respond to incentives, the evidence that contracts are designed in the way theory predicts is less convincing. Prendergast himself (2002) argued that the role of specific knowledge may explain why most empirical studies fail to find the negative risk-incentives relation predicted by standard theory.

In this section, I take Prendergast’s insight one step further. Empirical research on incentives has produced several puzzles other than the risk-incentives relation. All of the puzzles relate to predictions derived from the Informativeness Principle (Holmström 1979), and I argue that all of them can be explained by the role of specific knowledge. I first revisit the risk-incentives relation in the model studied here, and subsequently turn to predictions based on informativeness.

5.1 Uncertainty vs. Incentives Revisited

A central prediction of principal-agent theory is that a risk-neutral principal provides weaker incentives to a risk-averse agent the noisier the measure on which the agent’s compensation is based. However, empirical evidence of an inverse relation between risk and incentives is scarce. Although studies supporting this prediction (e.g. Aggarwal and Samwick 1999) are well known, Prendergast (2002) concludes that they are greatly outnumbered by studies finding a positive or insignificant relation between risk and incentives. This discrepancy between theory and evidence, also observed by Zabojnik (1996) and Lafontaine and Slade (2000), has attracted much interest in recent research.

One reason for the discrepancy may be that agents have knowledge about uncertain events and must be given incentives to use it. In Prendergast’s (2002) model, the agent

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20 See also Demsetz and Lehn (1985): “In less predictable environments, however, managerial behavior
has better information than the principal about the profitability of different available projects. Prendergast shows that an increase in the riskiness of the projects’ payoffs increases the value of the agent’s information and makes it more likely that the principal wants to delegate the choice of the project to the agent.

The present model formalizes somewhat more literally the idea that agents must be given incentives to respond to unforeseen events appropriately. It makes clear that whether one should expect to see a positive or a negative relation between incentives and risk depends on the source of uncertainty. According to Proposition 5, $e$ affects the optimal incentive contract and delegation profit in ways familiar from standard agency models (see also Footnote 12): the greater the noise in measuring output, the lower the optimal weight on output in the incentive contract, and the lower the principal’s delegation profit.

An increase in $t$, on the other hand, implies a greater value of the agent’s knowledge and increases the principal’s delegation profit (part c), which is similar to Prendergast’s (2002) main result. On its own, it leads to Prendergast’s prediction that risk and incentives are likely to be positively related if delegation decisions are not controlled for (e.g. if company-owned and franchised outlets of a chain are pooled), whereas controlling for delegation, they should be negatively related for the usual reasons. Parts (a) and (b) of the proposition, however, make clear that the picture is more complicated: even if the delegation of authority is taken as given, which is an appropriate assumption for most managerial occupations, the relation between incentives and uncertainty may be positive.

Two factors are responsible for the positive relation between technological uncertainty and (output-based) incentives: first, $\theta$ influences the agent’s individually optimal choice of effort. This condition is necessary because exposing the agent to risk is always costly for the principal and leads to lower-powered incentives unless the realized state of nature is relevant for agent’s actions. Second, the agent’s information pertains to the true productivity of his effort and thus affects his first-best optimal effort. Even though the agent cares about measured performance $y$ and not actual output $Y$, $\theta$ affects $y$ only through its effect on $Y$. Thus, the agent’s use of his private information always benefits the principal.

If, in contrast, the agent’s information pertains to measured performance instead of true productivity, it may affect the agent’s individually optimal action but not his first-best action. For instance, if the agent receives information that measured performance...
will be low, the agent has little incentive to exert effort even if the effect on the principal’s payoff is large. Baker (1992) and Baker, Gibbons and Murphy (1994) show that under this assumption, an increase in the variance of the objective performance measure may lead to lower optimal incentives placed on it. More generally, whether or not an agent’s use of his information is beneficial for the principal depends on the precise correlation structure between the agent’s information, the productivity of effort, and measured performance.\(^{21}\)

The above discussion suggests that there are serious obstacles to studying the empirical relation between incentives and risk, consistent with the current state of the literature. One problem is that it may be difficult to determine whether a manager’s knowledge pertains more to his measured performance or to true productivity, although one might expect that information about market conditions falls in the latter category. The main problem, however, is that while technological and environmental uncertainty are treated as independent parameters in this (reduced-form) model, in reality they are likely to be correlated. For example, for the CEOs of oil companies, oil price fluctuations may call for appropriate actions based on the CEOs’ expertise, but also affect the companies’ performance for reasons unrelated to any actions taken. Whether incentives ought to be negatively or positively related to a certain source of risk thus depends on how important responses to that risk are, which is bound to be difficult to determine for researchers\(^{22}\).

### 5.2 The Use of Aggregate Performance Measures

One implication of the Informativeness Principle is that if both noisy and less noisy measures of an agent’s performance are available, greater weight should be given to less noisy performance measures, other things equal. In terms of the input-output terminology used here, this means that preference should be given to input measures since (by definition)

\(^{21}\) See also Lizzeri, Meyer and Persico (2002), who show that providing interim performance evaluations to an agent are generally harmful for the principal when the agent’s productivity is known, and hence the evaluation only informs the agent about measured performance rather than socially optimal actions. In contrast, interim evaluations can be useful when there is uncertainty about the agent’s productivity.

\(^{22}\) An attempt to resolve this problem is offered by Shi (2005), who argues that firm- and industry-specific risk is more likely to be “respondable” than market-wide risk because CEOs are likely to have specific knowledge (relative to e.g. shareholders and analysts) about their firm and the industry, whereas they have little or no information advantage with regard to market risk. Consistent with the conjecture, Shi finds that CEO incentives are negatively related to market risk but positively to firm- and industry-specific risk. For a similar conjecture and results on managerial ownership, see Demsetz and Lehn (1985).
they are more closely related to the agent’s actions. 

This prediction is rarely borne out in the evidence. In a wide range of occupations, be it sales people, franchisees or executives, incentive pay is based largely on aggregate financial data and only partly on e.g. operational measures or other measures of “input”.

One reason for using output-based pay, of course, may be a lack of good input measures. If important dimensions of an agent’s performance are difficult to measure, dysfunctional responses are likely if compensation is based on more easily available input measures, see Holmström and Milgrom (1991). Extending this argument, Baker (2002) has pointed out that principals often face a tradeoff between noisy performance measures that are closely correlated with the principal’s payoff (i.e. output measures) and less noisy but distorted input measures. It follows that unless the agent’s input can be measured rather well, there may be little choice but to base compensation on output measures.

And yet this does not seem to be the whole story if we want to understand the prevalence of output-based pay. Firms nowadays often have an abundance of information that about the minutiae of their employees' activities, through the use of activity-based costing, balanced scorecards, and integrated information systems. While multitask measurement problems continue to matter in many occupations, substantial progress has been made in devising ways to measure performance in almost all relevant dimensions of, for instance, sales people or managers.

But even firms that go to great trouble to measure employees’ inputs find it difficult to use these measures in their compensation plans. The main problem is that of determining the optimal weights for each measure in a compensation plan. As it turns out, firms simply do not know very well how their employees' actions contribute to firm performance, whereas the employees often have an information advantage. The best option then is to base compensation on the principal’s objective rather than measures of the agent’s actions (see Murphy, 1999, and Ittner et al., 2003).

The present model captures precisely this problem, and Proposition 3 confirms the above conclusion: even if effort can be observed without noise, partly output-based compensation is optimal, and its weight increases with the importance of the agent’s specific knowledge. The paper thus explains why firms base their managers’ compensation on aggregate financial measures even when detailed measures of managerial activity are available.
5.3 Paying Agents for “Luck”

The Informativeness Principle implies that an optimal incentive contract uses all information that leads to a more precise measurement of the agent’s effort. In particular, if a performance measure is influenced by factors outside the agent’s control (such as fluctuations in demand), these influences should be filtered out of the performance measure in order to obtain a more precise measure of the agent’s effect on it.

In a well-known paper, Bertrand and Mullainathan (2001) document that this prediction appears to violated in the U.S. oil industry. They observe that the performance of oil companies varies systematically with the market price of oil, which in turn is easy to measure. Theory would then predict that the influence of the oil price on firm performance should be filtered out of the performance measure on which a CEO’s compensation is based. Instead, Bertrand and Mullainathan find that CEO compensation responds as much to changes in firm performance due to oil price shocks as to general changes in firm performance. They interpret this finding as evidence of the CEOs’ power to extract rents from shareholders, an idea that has gained some popularity recently (see Section 6, point 8).

Instead, I argue that Bertrand and Mullainathan’s findings can be explained by incorporating specific knowledge into the theory of incentives, without resorting to an altogether different explanation based on governance failure. To make this argument more precise, I now examine how optimal incentives are affected when the principal has information about the uncertain production technology \( \theta \).

To simplify the analysis and facilitate a comparison with standard agency models, I assume here that there is only one task (as in Section 3), and that compensation can be based only on output. Suppose expected output is given by \( E[Y] = a\theta \), with \( \theta \in \{\theta_H, \theta_L\} \). If the agent knows \( \theta \) but the principal does not, the optimal piece rate \( \gamma^* \) is given by Proposition 2 for \( k = 1 \). If, in contrast, the principal can condition the contract on information about \( \theta \), she will choose a piece rate \( \gamma_H \) if \( \theta = \theta_H \), and \( \gamma_L \) if \( \theta = \theta_L \), with \( \gamma_H > \gamma^* > \gamma_L \). Thus, compared to the case of an uninformed principal, the agent will face stronger incentives in good states of the world and weaker incentives in bad states. These incentives give rise to a convex pay-performance relation, consistent with the evidence of Garvey and Milbourn (2003).

Now suppose that the productivity of effort is negatively related to \( \theta \). Of course, in the model they are positively related (\( \theta \) is the productivity), which is without loss of
generality since values of $\theta$ can always be appropriately relabeled. For any given proxy of $\theta$, however, the relation is not so obvious. For instance, if $\theta$ is a measure of market demand, it depends on the particular circumstances whether the value of effort is highest when demand is high, or when it is low. If optimal effort is negatively related to $\theta$, then incentives are strongest in the bad state of the world, which is the opposite of the conclusion above.

If the principal does not know how optimal effort and a given $\theta$ are related, she cannot make use of information about $\theta$ at all. The same is true when different states of the world call for different optimal actions, but in a way that cannot be associated with “high” or “low” effort. Specifically, suppose that expected output is given by $a_1 \theta_1 + a_2 \theta_2$ as in the main model, with $\rho = -1$. In this case, (first-best-)optimal effort takes only two values, depending on whether $\tau = (1, -1)$ or $\tau = (-1, 1)$. Again, there is no scope for using information about $\theta$ in the incentive contract, unless compensation can be conditioned on input measures as well.

Ideally, the principal would still like to insure the agent against income risk resulting from fluctuations in $\theta$, but the scope for doing so is very limited. In the present setting, for instance, the principal might want to include a term $-\delta \theta$ with $\delta > 0$ in the agent’s wage function. But with limited liability that is not possible since the agent’s main bonus $\gamma y$ can always end up being zero due to bad luck; the same is true if environmental risk and technological uncertainty are correlated. Thus, the only way for the principal to insure the agent against risk is through partially input-based pay. In a model with risk aversion instead of risk neutrality with limited liability, these conclusions would probably be less stark but similar.

The above discussion implies that observable shocks outside the agent’s control will not be filtered out of the performance measure if the agent is expected to respond to them. Or, in the language of Bertrand and Mullainathan (2001), compensation will generally depend as much on a “lucky dollar” as on a “general dollar”, just as Bertrand and Mullainathan find.\(^{23}\) As I discuss below in Section 6, the present model may also explain their finding that firms are more likely to reward a “lucky dollar” the worse they are governed.

\(^{23}\) Bertrand and Mullainathan argue that if specific knowledge were the explanation, one would expect to see only unusually successful CEOs rewarded for luck, contrary to the data. But this counterargument does not hold in the present model, since even if the average CEO is no better able than others to forecast changes in market conditions, he is still likely to have specific knowledge of how best to respond to them.
5.4 The Scarce Use of Relative Performance Evaluation

As mentioned above, the Informativeness Principle implies that factors outside an agent’s control should be filtered out of his performance measure. Often these factors are best measured by using the performance of other agents as a benchmark, which is the basic logic of relative performance evaluation (RPE), see Holmström (1982). A puzzle that has intrigued empirical researchers for quite some time now is why RPE appears to be used to a much lesser extent in practice than theory would suggest (Prendergast, 1999; Murphy, 1999). I argue that, once again, the role of specific knowledge may explain the scarce use of RPE; the argument is very similar to that of Section 5.3.

In the case of pure technological risk, the conclusions of Section 5.3 apply without change. When the performance of both firm A and firm B is positively related to $\theta$, then B’s performance can be used as a proxy for $\theta$. When in addition optimal effort in both firms is positively related to $\theta$, then firm A’s optimal contract is one in which the incentives for A’s agent (not just the level) vary positively with B’s performance. The reverse holds when optimal effort in both firms is negatively related to $\theta$. Finally, when optimal actions depend on $\theta$ but overall effort and performance do not, then information about B’s performance is of no use to A.

These observations imply that the scope for RPE is very limited when agents’ performances are linked through events to which agents the must respond. In reality, technological and environmental risk are likely to be correlated as they may be driven by the same random events. Overall, then, the usefulness of RPE therefore depends on the importance of agents’ responses, or in the terminology of this paper, to what extent uncertainty is technological or environmental.24

6 Implications and Predictions

Proposition 5 leads to many empirical predictions and other implications. The predictions relate incentive contracts and delegation decisions to characteristics of the agent’s job, the relationship between agent and principal, properties of the firm or the firm’s environment, and are discussed below in that order. Since the main novelty of the paper lies in pointing

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24 See also Meyer and Vickers (1997), who point out that in the presence of career concerns, the usefulness of RPE depends on the relative size of the correlations of time-invariant and transient influences on agents’ performances.
out the role of specific knowledge for incentive contracting (whereas it is familiar for delegation decisions), I place a greater emphasis on predictions about incentives and performance measurement in what follows.

Most predictions are derived from different interpretations of the knowledge parameter $k$. Note that $k$ measures the agent’s knowledge relative to that of the principal. Thus, Proposition 5 implies that delegation and the relative weight on output-based pay are positively related to the gap between agent and principal in knowledge pertaining to the agent’s job.25

1. Knowledge gap: The results in Proposition 5 for $k$ imply that we should expect to see stronger incentives, a greater weight on output-based pay, and a greater extent of delegation, the better the agent’s specific knowledge about his job is relative to the principal’s knowledge. One way to test this prediction is to directly attempt to measure the gap in job-specific knowledge between agent and principal. This has been done with some success in accounting research using survey data. For instance, Abernethy et al. (2004) and Bouwens and van Lent (2005) use a six-item scale developed by Dunk (1993) to measure the extent of information asymmetry between profit center managers and their superiors.26

The measure obtained is strongly correlated with “harder” proxies of specific knowledge such as the manager’s experience (in years) in his current position, his age, and the ratio of industry experience between manager and superior (the difference in education backgrounds would be another possible proxy). Abernethy et al. (2004) find that delegation is more likely the greater the informational asymmetries, consistent with Proposition

25 It is more difficult to make predictions about the overall strength of incentives because that will depend on the principal’s and agent’s absolute, not just relative, levels of knowledge. For example, an increase in the principal’s knowledge should in the context of the present model be interpreted as a decrease in $k$, leading to shift towards input-based pay. On the other hand, it is possible that a better informed principal would also want to provide stronger overall incentives to the agent, which may explain the apparent contradiction between Proposition 5 and the result of Baiman, Larcker and Rajan (1995) that “the compensation risk imposed on the business manager generally increases with the principal’s relative expertise” (p.206).

26 This involves literally including questions on the survey such as “Compared to your superior, who is more familiar with the input-out relationships inherent in the internal operations of your organizational unit?” and “Compared to your superior, who is more familiar technically with the work of your organizational unit?”
5 (c). Bouwens and van Lent (2005) find that the weight in managers’ compensation placed on aggregate accounting measures of division profit (=output), as opposed to disaggregated metrics or non-financial measures (=inputs), increases with measures of managers’ authority. A untested prediction is that even controlling for managers’ authority, the weight on output-based pay should increase with the extent of informational asymmetries. For other ways to measure specific knowledge directly, see Christie, Joye and Watts (2003)).

2. Job complexity: Several empirical researchers have studied the effects of job complexity on e.g. the level and structure of compensation.\(^{27}\) In the model, job complexity is captured both by \(k\) and by \(\rho\). First, the more complex the agent’s job is, the better the agent probably knows what actions are value-maximizing relative to the principal, which corresponds to a higher value of \(k\). Second, the smaller \(\rho\), the more likely the \(\theta_i\) and hence the efficient effort levels for each task will be different. A job with smaller \(\rho\) is thus more complex in the sense that the optimal allocation of total effort across tasks is more difficult to specify in advance. Both ways of thinking about job complexity lead to the same prediction that the weight on output-based incentives and the principal’s profit under delegation should vary positively with measures of job complexity.\(^{28}\) The findings of Ortega (2003) and Christie, Joye and Watts (2003) confirm this prediction for delegation decisions. The relation between job complexity and the choice of performance measures has to my knowledge not yet been investigated.

It can be shown that parameter changes leading to more output-based pay also lead to a higher expected wage for the agent. We then obtain the prediction that wages are positively related to job complexity, consistent with the evidence of Van Ophem et al. (1993) and Pekkarinen (2002). To explain this, Van Ophem et al. (1993) argue that a more complex job requires greater effort or in other ways increases a worker’s disutility, for which a worker needs to be compensated. Here, instead, a wage increase associated with greater complexity results from a greater reliance on output-based pay, whereas (as


\(^{28}\) This definition of complexity differs from Kremer’s (1993), who assumes that tasks are complementary and measures complexity by the number of tasks. In particular, here the tasks are independent in the sense that the cross-partial derivatives of \(a_1\) and \(a_2\) in the expected value of \(Y\) are zero. My conjecture, though, is that in a suitably extended model with \(n\) tasks, \(\gamma^*\) and the delegation profit would also be positively related to \(n\).
numerical simulations suggest) the direct effect of complexity on the agent’s utility before adjusting the contract is only small.

3. Subjective performance evaluations: A common way for firms to deal with distortions created by the use of objective performance measures is to base e.g. bonus payments on subjective performance evaluations. The usefulness of such evaluations, however, hinges on the ability of superiors to evaluate their subordinate managers’ performance. But if managers have specific knowledge about how to do their job, their superiors’ evaluations are likely to be based more on observable actions than on the managers’ true contributions to the firm. Subjective evaluation then becomes an input measure, in contrast to its role in, say, Baker, Gibbons and Murphy (1994), where it is assumed to produce an unbiased measure of output.

Under this new interpretation, the purpose of subjective evaluations is to shield a manager from the noise contained in other, objective performance measures (“your unit had a bad year but we know you worked hard”. Consistent with this view, Gibbs et al. (2003a) find that firms are more likely to pay bonuses based on subjective evaluation in periods with losses.

Thinking of subjective evaluations as input measures also leads to predictions about the use of subjective vs. objective performance measures. For example, it is plausible that the knowledge gap between CEOs and boards in a company is greater than that between lower-level executives and the superiors who evaluate them. One then obtains the prediction that subjective evaluations are more likely to be used for lower-level executives than CEOs, which is what Murphy and Oyer (2003) find (their own, also plausible, explanation is a lack of reliable objective measures at lower levels). The limited usefulness of subjective evaluations when evaluators are less well informed than their agents may also explain the prevalence of objective measures in executive compensation plans.

4. Congruence: The parameter \( \phi \) measures how substitutable the two tasks are from the agent’s point of view. According to Propositions 4 and 5, a higher value of \( \phi \) is associated with a higher value of the agent’s information and accordingly stronger and more output-based incentives and a greater likelihood of delegation.

A way to interpret this result is that the larger \( \phi \), the better the agent’s and the principal’s interests are aligned since the agent is more likely to allocate his effort in the way the principal would prefer (if she had the same information). The role of \( \phi \) in this model is thus reminiscent of that of “congruence” parameters in delegation models such
as those of Aghion and Tirole (1997) and Dessein (2002), which measure the probability with which principal and agent prefer the same project. Part (c) of Proposition 5 for \( \phi \) is in this sense similar to Dessein’s result that delegation is more likely the more congruent the agent’s preferences are with the principal’s.

5. Value of effort: Standard models of incentive contracting predict, like the result of Proposition 5 (a), that optimal incentives are increasing in the value of managerial effort \( \theta \). In addition, Proposition 5 predicts that a higher value of effort is associated with a higher likelihood of delegation and, conditional on delegation, a greater weight on output-based pay. Different ways to measure the value of effort are discussed in Lafontaine and Slade (2000) in the context of franchise decisions. These include measures of labor intensity, a manager’s experience in the industry, or in retail gasoline, a measure distinguishing full from self service. In all studies reported that find a statistically significant relation between the measure chosen and the franchise decision, the value of managerial effort has a positive effect on the decision to franchise out a business unit.

6. Group-based vs. individual incentives: Employees such as division managers agents are often compensated based on their own unit’s performance but also the performance of the group (which may be the entire firm) they are part of; see Ittner and Larcker (1998) and Murphy (1999). Between these two types of incentives, measures of individual performance are inputs, while group performance measures output. Group-based incentives are typically interpreted as an optimal response to externalities between different units of a firm, see e.g. Bushman, Indjejikian and Smith (1995) and Keating (1997) for a discussion and empirical analysis.

The present model suggests an alternative motivation for group-based pay. Even if individual (unit) performance can be measured well, and even in the absence of externalities, group-based pay is optimal if a manager has specific knowledge about how his unit contributes to group performance.\(^{29}\) Parts (a) and (b) of Proposition 5 can then be used to derive predictions about the relative weights of individual vs. group incentives in managers’ compensation plans.

For instance, interpreting job complexity as a proxy for \( k \) (see above), it follows that the weight on group-based incentives should be increasing in job complexity, while the weight

\(^{29}\) This argument can be developed formally in a variation of the main model in which two managers exert effort on one task each to produce a group output. For details, contact the author for a previous version of the paper.
on individual incentives should be decreasing. That is precisely what Ortega (2003) finds in his study of the determinants of incentive compensation from survey data on European employees. As Ortega points out, the first result is consistent with a standard agency model, but the second is not.

7. Growth options: Smith and Watts (1992) have suggested that both the level of compensation and the incentives provided to executives should be positively related to a firm’s growth opportunities. They argue that the greater the growth options, the less shareholders are able to evaluate a CEO’s decisions, and hence the more they will want to tie compensation directly to firm value by placing a larger weight on stock-based pay.

The present model cannot shed light on Smith and Watts’ basic hypothesis, but offers an analytical foundation of their conclusion. Under Smith and Watts’ hypothesis, better growth options are associated with a larger value of $k$ (or $t$) since $k$ measures the knowledge gap between shareholders and a CEO. We then obtain the prediction that firms with better growth options place a relatively larger weight on stock-based pay, consistent with the evidence of Smith and Watts (1992) and Gaver and Gaver (1993). For similar evidence on delegation decisions, see Christie, Joye and Watts (2003). Conversely, older firms and firms in more mature industries should be expected to place relatively less emphasis on stock-based pay due to a smaller knowledge gap between board and CEO.

8. Corporate governance: A substantial amount of recent research examines the connections between corporate governance and CEO compensation. A new view has become popular that holds that CEO pay is determined in a process in which, as it were, the roles of board and CEO as principal and agent are reversed (see Bebchuk and Fried, 2004). A growing body of evidence documents that the worse companies are governed (using a variety of measures), the more money their CEOs make (see e.g. Core et al., 1999, or Bertrand and Mullainathan, 2001). This has been interpreted as evidence that CEOs are able to extract rents from shareholders when directors lack the ability or incentive to exert stronger control, or worse, when they believe to owe the CEO a favor in return for their appointment to the board. In support of this view, its proponents argue that if conventional theory were valid, we should not expect to see strong relations between the quality of governance and the level and structure of CEO pay.

In contrast, the present model makes clear that once one recognizes that CEOs typically have specific knowledge about how to do their job, it is very natural that both structure and level of CEO compensation would depend on the quality of corporate gov-
ernance. Specifically, the results of Proposition 5 for \( k \) imply that the greater is the knowledge gap between a CEO and the company’s board, the stronger the CEO’s incentives and the larger the weight on output (=stock)-based pay will be. As mentioned above (see point 2. in this section), a larger value of \( k \) is also associated with a higher expected wage for the agent. The knowledge gap, in turn, depends on the characteristics of both principal and agent.

It follows that CEO pay *should* vary with measures of corporate governance to the extent they are correlated with how well the board understands (or has an incentive to understand) the CEO’s job. Core et al. (1999), for instance, find that the level of CEO pay is positively related to the share of outsiders on the board. This finding is somewhat unexpected from the perspective of the “managerial power” view outlined above, since outsiders on the board are less likely to owe the CEO a favor, which is normally interpreted as leading to more effective governance. A different interpretation, however, is that outside directors also understand the firm less well, leading to a larger knowledge gap between board and CEO.\(^{30}\)

The purpose of this discussion is not to argue against the managerial power view of CEO compensation. Instead, it is to show that the theory presented here predicts connections between corporate governance and CEO compensation previously considered outside the domain of the theory of incentives. These connections include some that have been cited in support of an altogether different view of the process by which CEO pay is determined. More refined tests than previously employed, therefore, are necessary to distinguish the two contrasting views on how CEO pay is determined.

9. **Product market competition:** The value of managerial effort \( \bar{\theta} \) depends in part on a firm’s competitive environment. In Raith (2003), I argue that greater product market competition implies a higher marginal value for firms of improving their competitive position (relative to others) through cost reductions or quality improvements. Since managerial effort is required to implement such changes, greater competition therefore implies a higher value of managerial effort. Proposition 5 then predicts that, other things equal, greater product market competition should be associated with more output-based

\(^{30}\) Two broad trends, not previously connected, are also consistent with the argument that governance and CEO pay are linked through specific knowledge: a trend towards better corporate governance, brought about for instance by greater outsider presence on boards (cf. Hermalin 2003, Murphy and Zabojnik 2003); and a trend towards greater use of non-financial performance measures (cf. Ittner and Larcker 1998)
pay and a greater extent of delegation. Consistent with these predictions, Slade (1998) finds that oil companies are more likely to delegate pricing decisions to gas stations the greater the price and cross-price elasticities of demand. Kole and Lehn (1999) find that deregulation in the U.S. airline industry was followed by substantial increases in CEO pay as well as a shift towards more stock-based compensation. More directly pertinent to the prediction of Proposition 5, Karuna (2005) finds in a cross-sectional study that more competitive industries provide stronger stock-based incentives relative to earnings-based incentives.

10. Technological uncertainty: Previous research on the relation between uncertainty and incentives has sought to find evidence for the negative relationship predicted by standard theory; findings of a positive relation were seen as an anomaly. An alternative and novel approach would be to do the opposite; i.e., to examine whether incentives and delegation are positively related to proxies of technological uncertainty ($t$ in the model).

A candidate for such a proxy is the concept of “industry clockspeed” developed by Mendelson and Pillai (1998) as a quantitative measure “that gauges the velocity of change in the external business environment and sets the pace of ... firms’ internal operations”. It is a composite of three variables: product life (the duration of the product life cycle), product “freshness”, measured by the share of total revenues due to products introduced over the previous 12 months, and the rate of change in input prices. Although all of these variables affect both technological and environmental risk, they are highly decision-relevant, suggesting that clockspeed may be a suitable proxy for $t$. We then obtain the prediction that the strength of incentives, the weight on output measures and the likelihood of delegation should be positively related to clockspeed. Evidence suggesting that clockspeed has a positive effect on the value of decentralized decision-making is reported in Mendelson (2000).

7 Conclusion

Standard agency models assume that a principal cannot observe her agent’s actions but knows what these actions should be. Yet in many of the most important occupations, the problem is the opposite one: “the reason shareholders entrust their money to selfinterested CEOs is based on shareholder beliefs that CEOs have superior skill or information in making investment decisions” (Murphy 1999, p.2521), whereas the observability of those
decisions is a less salient problem. Incorporating specific knowledge in the theory of incentives has been difficult because extensions of workhorse principal-agent models quickly prove to be analytically intractable.

A different strand of the literature recognizes the importance of an agent’s knowledge, but focuses almost entirely on the question of when a principal should delegate tasks to an agent (see the references in footnote 2), while ignoring incentives. Although both the theory of incentives and the literature on delegation shed light on agency relationships within firms, their perspectives on them are very different and without overlap.

This paper combines both perspectives. I study the effects of specific knowledge on optimal incentives, performance evaluation and delegation in a model that leads to simple, closed-form solutions and intuitive comparative statics. The main formal result is that when an agent has specific knowledge about how his actions contribute to the principal’s objectives, the principal must provide incentives based on “output” measures correlated with those objectives rather than on measures correlated with the agent’s actions.

I develop two sets of implications. First, in the model, the Informativeness Principle of standard agency theory no longer holds. It follows that all predictions based on the informativeness principle may fail empirically when agents have specific knowledge. I illustrate this conclusion with reference to several puzzles in the empirical literature, and argue that all of them can be resolved with the same explanation. Second, the formal results lead to a wide range of empirical predictions and other implications. Some of the predictions are already supported by evidence, some suggest a reinterpretation of previous findings, and some are yet to be tested. In particular, the paper suggests an “input-output” distinction as a new way to categorize performance measures, which in conjunction with a focus on the role of specific knowledge will hopefully prove fruitful both for empirical research and the practical design of compensation plans.
Appendix: Proofs

Proof of Proposition 1: The principal’s expected profit conditional on \((\theta, s)\) is

\[
\pi = Y(\theta, s) - w(\theta, s) = Y(\theta, s) - \beta a(s) - \gamma \left[ \frac{e}{2} + (1 - e)Y(\theta, s) \right]
\]

\[
= [1 - \gamma(1 - e)]Y(\theta, s) - \beta a(s) - \frac{\gamma e}{2}.
\]  

(16)

We want to compute the expected value of (16) over \((\theta, s)\). For \(Y\), we have

\[
Y(\theta, s) = a(s)\theta = [\beta + (1 - e)\gamma \hat{\theta}(\theta, s)]\theta = \beta \theta + (1 - e)\gamma \hat{\theta}(\theta, s)\theta.
\]  

(17)

From \(\theta = 1 + t\tau\) and \(\hat{\theta} = 1 + kts\), we have \(E(\hat{\theta}\theta) = E[(1 + kts)(1 + t\tau)]\). Using \(E(s) = E(\tau) = 0\), this expression simplifies to \(1 + kt^2E(\tau s)\). Finally, we have \(\tau s = 1\) if \(\tau = s\) and \(-1\) otherwise; and the relation \(Pr(s = \tau) = (1 + k)/2\) implies that the prior probabilities of the events \(\tau = s = \pm 1\) are \((1 + k)/4\) while the probabilities of events \(\tau \neq s\) are \((1 - k)/4\). Therefore,

\[E(\tau s) = 2 \frac{1 + k}{4} \frac{1}{1 + 2 \frac{1 - k}{4} (-1)} = k.\]

Putting everything together, the expected value of (17) is

\[
\beta + (1 - e)(1 + k^2t^2)\gamma.
\]  

(18)

Moreover, using (4), the expected value of \(a(s)\) is simply \(\beta + (1 - e)\gamma\). Substitute this expression and (18) into (16) to obtain the principal’s expected profit:

\[
E(\pi) = [1 - \gamma(1 - e)][\beta + (1 - e)\gamma(1 + k^2t^2)] - \beta[\beta + \gamma(1 - e)] - \frac{\gamma e}{2}.
\]  

(19)

Differentiating (19) with respect to \(\beta\) and \(\gamma\) leads to the first-order conditions

\[
1 - 2(1 - e)\gamma - 2\beta = 0 \quad \text{and} \quad -\frac{e}{2} + (1 - e)[2(1 - e)\gamma](1 + k^2t^2) - 2(1 - e)\beta = 0
\]  

(20)

(21)

The solution of (20) and (21) for \(\beta\) and \(\gamma\) is stated in the proposition. It is straightforward to establish that (19) is strictly concave in \(\beta\) and \(\gamma\) and so the solution of (20) and (21) is indeed a maximum. Moreover, it is straightforward to verify that \(\gamma^*\) in (9) is positive if and only if Condition (a1) holds.

Finally, we need to make sure \(\theta a\) does not exceed 1. It attains its largest value when \(\tau = s = 1\). Using (4) and \(\hat{\theta} = (1 + kts)\), we obtain

\[
Y(1, 1) = \frac{(1 + t)[\beta + (1 - e)\gamma(1 + kt)]}{4(1 - e)kt}
\]

(22)

35
Subtracting the numerator in (22) from the denominator leads to the left-hand side of Condition (a3). Thus, under (a3), (22) does not exceed 1.

Proof of Proposition 2: The agent’s individually optimal effort vector is given by (4) with \( \beta \) set to 0. Consequently, the principal’s profit is also given by (19) with \( \beta \) set to 0:

\[
E(\pi) = [1 - \gamma(1 - e)](1 - e)\gamma(1 + k^2t^2) - \frac{\gamma e}{2}.
\]

Differentiation with respect to \( \gamma \) leads to the stated expression. Parameter conditions similar to (a1) and (a2) guarantee an interior solution.

Proof of Proposition 3: In stage 4 of the game, the agent chooses his effort \( a \) as a function of beliefs \( \hat{\theta} \) about \( \theta \). Assuming an interior solution for all endogenous variables, the agent’s expected utility is

\[
\beta(a_1 + a_2) + \gamma\{e/2 + (1 - e)[\hat{\theta}_1a_1 + \hat{\theta}_2a_2]\} - \frac{d}{1 + \phi} \left( a_1^2 + a_2^2 + 2\phi a_1a_2 \right),
\]

(23)

cf. (3). This expression is strictly concave in \( a \), and maximization with respect to \( a \) leads to

\[
a_i^*(\hat{\theta}) = \frac{\beta}{2d} + \frac{(1 - e)(\hat{\theta}_i - \phi\hat{\theta}_j)\gamma}{2d(1 - \phi)}
\]

(24)

for \( i = 1, 2; j \neq i \), which is an affine function of \( \hat{\theta} \).

Let us next determine \( \hat{\theta} \). For \( \tau_1 \in \{-1, 1\} \), we have \( \tau_1\tau_2 = 1 \) if \( \tau_1 = \tau_2 \) and \( \tau_1\tau_2 = -1 \) otherwise. Since \( \Pr(\tau_1 = \tau_2) = (1 + \rho)/2 \), it follows that for \( \tau \in \{-1, 1\}^2 \), we have \( \Pr(\tau) = (1 + \rho\tau_1\tau_2)/4 \). Moreover, since \( \Pr(s_i = \tau_i) = (1 + k)/2 \) and since the conditional distributions \( s_i|\tau_i \) are independent, it follows that for any \( (\tau, s) \in \{-1, 1\}^4 \), the probability of \( s \) conditional on \( \tau \) is given by \( \Pr(s|\tau) = (1 + ks_1\tau_1)(1 + ks_2\tau_2)/4 \). The unconditional probability of \( s \in \{-1, 1\}^2 \) is then given by \( \Pr(s) = \sum_{\tau \in \{-1, 1\}^2} \Pr(s|\tau)\Pr(\tau) \), which simplifies to \((1 + k^2\rho s_1s_2)/4 \). The expected value of \( \tau \) conditional on \( s \) is therefore given by

\[
E(\tau|s) = \sum_{\tau \in \{-1, 1\}^2} \tau \frac{\Pr(s|\tau)\Pr(\tau)}{\Pr(s)},
\]

which reduces to

\[
E(\tau_i|s) = \frac{k(s_i + \rho s_j)}{1 + k^2\rho s_1s_2}
\]

for \( i = 1, 2 \) and \( j \neq i \). We therefore have

\[
\hat{\theta}_i(s) = \left[ 1 + t \frac{k(s_i + \rho s_j)}{1 + k^2\rho s_1s_2} \right] \bar{\theta} \quad \text{for } i = 1, 2 \text{ and } j \neq i.
\]

(25)

In particular, for \( k = 0 \) we have \( \hat{\theta}_i = \bar{\theta} \), whereas for \( k = 1 \), \( \hat{\theta}_i = \theta_i \) irrespective of \( \rho \) and \( s_j \), which follows because \( s_i \in \{-1, 1\} \).
As in the one-task model, what remains is to substitute (24) and (25) into the principal’s profit function and take expectations over all joint realizations of \( \tau \) and \( s \). The principal’s expected profit conditional on \((\theta, s)\) is

\[
\pi = Y(\theta, s) - w(\theta, s) \\
= Y(\theta, s) - \beta(a_1(s) + a_2(s)) - \gamma \left[ \frac{e}{2} + (1 - e)Y(\theta, s) \right] \\
= [1 - \gamma(1 - e)]Y(\theta, s) - \beta(a_1(s) + a_2(s)) - \frac{\gamma e}{2}.
\]

(26)

To obtain the expected value of (26) over \((\theta, s)\), we need to evaluate the expected values of \( a_1(s) + a_2(s) \) and \( Y(\theta, s) \). First, notice from (24) that

\[
a_1 + a_2 = \frac{1}{2d} \{2\beta + (1 - e)[\hat{\theta}_1(s) + \hat{\theta}_2(s)]\gamma\},
\]

where the expected value of \( \hat{\theta}_1(s) + \hat{\theta}_2(s) \) over \( s \) is \( 2\bar{\theta} \). Hence, the expected value of \( a_1 + a_2 \) in (26) over \( s \) is given by

\[
\frac{\beta + \gamma(1 - e)\bar{\theta}}{d}.
\]

(27)

Next, we have

\[
Y(\theta, s) = a_1^*(s)\theta_1 + a_2^*(s)\theta_2 \\
= \frac{\beta}{2d}(\theta_1 + \theta_2) + \frac{(1 - e)\gamma}{2d(1 - \phi)} \left[ \theta_1(\bar{\theta}_1 - \phi\bar{\theta}_2) + \theta_2(\bar{\theta}_2 - \phi\bar{\theta}_1) \right]
\]

(28)

(arguments of \( \hat{\theta}_i \) omitted). The expected value of \( \theta_1 + \theta_2 \) in the first term is \( 2\bar{\theta} \). Obtaining the expected value of the term in \( \{ \} \) requires evaluating the term for each of the 16 permutations of \((\theta_1, \theta_2, s_1, s_2)\) and weighting it with the corresponding probability of the permutation, given by \((1 + ks_1t_1)(1 + ks_2t_2)(1 + \rho t_1 t_2)/16\). The computation is complex, but the result is very simple, it is \( 2(1 - \phi + k^2t^2\eta)\bar{\theta}^2 \), with \( \eta \) as defined in (8). Thus, the expected value of (28) is

\[
\frac{\bar{\theta} \beta}{d} + \frac{(1 - e)\gamma\bar{\theta}^2}{d(1 - \phi)}(1 - \phi + k^2t^2\eta)
\]

(29)

Substitute (27) and (29) into (26) to obtain the principal’s expected profit:

\[
E(\pi) = [1 - \gamma(1 - e)] \left[ \frac{\bar{\theta} \beta}{d} + \frac{(1 - e)\gamma\bar{\theta}^2}{d(1 - \phi)}(1 - \phi + k^2t^2\eta) \right] - \beta \frac{\beta + \gamma(1 - e)\bar{\theta}}{d} - \frac{\gamma e}{2}.
\]

(30)

Differentiating (30) with respect to \( \beta \) and \( \gamma \) leads to the first-order conditions

\[
\frac{[1 - 2(1 - e)\gamma]\bar{\theta} - 2\beta}{d} = 0 \quad \text{and} \quad \frac{[1 - 2(1 - e)\gamma]\bar{\theta} - 2\gamma}{d} = 0.
\]

(31)
\[ -\frac{e}{2} + \frac{(1 - e)\bar{\theta} \left[ (1 - 2(1 - e)\gamma)(1 - \phi + k^2t^2\eta)\bar{\theta} - 2(1 - \phi)\beta \right]}{d(1 - \phi)} = 0 \]  

The solution of (31) and (32) for \( \beta \) and \( \gamma \) is stated in the proposition. Since

\[ \frac{\partial^2 E(\pi)}{\partial \beta^2} = -\frac{2}{d} < 0, \quad \frac{\partial^2 E(\pi)}{\partial \gamma^2} = -\frac{2(1 - e)^2(1 - \phi + k^2t^2\eta)\bar{\theta}^2}{d(1 - \phi)} < 0, \quad \text{and} \]

\[ \frac{\partial^2 E(\pi)}{\partial \beta^2} \frac{\partial^2 E(\pi)}{\partial \gamma^2} - \left( \frac{\partial^2 E(\pi)}{\partial \beta \partial \gamma} \right)^2 = \frac{4(1 - e)^2\eta k^2t^2\bar{\theta}^2}{d^2(1 - \phi)} > 0, \]

\( E(\pi) \) is strictly concave in \( \beta \) and \( \gamma \) and so the solution of (31) and (32) is indeed a maximum. What remains to be shown is that under (A1)-(A3), the interior solution obtained here is indeed valid. Three conditions must be satisfied:

1. The expression for \( \gamma^* \) in (9) must be positive; otherwise we obtain a corner solution with \( \gamma^* = 0 \). It is straightforward to verify that this condition is equivalent to (A1).

2. The agent’s effort must be positive for each realization of \( s \) for (24) to be valid. There are two candidates for the lowest value of \( a_1(s) \), namely \( a_1(-1, 1) \) and \( a_1(-1, -1) \). Using (24) and (25), the sign of the difference \( a_1(-1, -1) - a_1(-1, 1) \) can be computed as

\[ \phi(1 - k^2\rho^2) - \rho(1 - k^2). \]  

(33)

If (33) is negative, the smallest value of \( a_1 \) is given by

\[ a_1(-1, -1) = \frac{(1 + k^2\rho)\beta + (1 - e)\gamma(1 - k)(1 - k\rho) + k(1 - t)(1 + \rho)}{2d(1 + k^2\rho)}, \]

which is always positive. If (33) is positive, the smallest value of \( a_1 \) is given by \( a_1(-1, 1) \). Using (24) and substituting for \( \bar{\theta} \) from (25) and for \( \beta \) and \( \gamma \) from (9), one can verify that the condition for \( a_1(-1, 1) \) to be positive is given by (A2).

3. Finally, \( \theta_1a_1 + \theta_2a_2 \) must not exceed 1. The largest value of \( \theta_1a_1 + \theta_2a_2 \) is attained when \( \tau = s = (1, 1) \), and using (24) and (25) leads to

\[ Y((1, 1), (1, 1)) = \frac{(1 + t)\bar{\theta}}{d} \left\{ \beta + (1 - e)\frac{[1 + k^2\rho + (1 + \rho)kt]\bar{\theta}\gamma}{1 + k^2\rho} \right\}, \]

which is less than 1 as long as

\[ d(1 + k^2\rho) > (1 + t)\bar{\theta}(1 + k^2\rho)\beta + (1 - e)(1 + k^2\rho + (1 + \rho)kt)\bar{\theta}\gamma). \]  

(34)

Substituting for \( \beta \) and \( \gamma \) in (34) from (9), and using (8), leads to condition (A3).

**Proof of Proposition 4:** 1. Substituting (11) into \( E[Y] = a_1\theta_1 + a_2\theta_2 \), the expected output is given by

\[ \frac{1}{2d(1 - \phi)} \left[ \theta_1(\bar{\theta}_1 - \phi\bar{\theta}_2) + \theta_2(\bar{\theta}_2 - \phi\bar{\theta}_1) \right]. \]  

(35)
The term in square brackets in (35) is again 2(1 − ρ) since (using (8)) the derivative of θ over θ is

The term in square brackets in (36) is the same as the one in (28), whose expected value over θ and s was already determined as 2(1 − φ + k^2t^2η)̃θ^2 in the proof of Proposition 3.

Next, using (11), into the agent’s disutility \( \frac{a}{1+\phi}(a_1^2 + a_2^2 + 2\phi a_1 a_2) \) as a function of first-best effort is given by

\[
\frac{\theta^2}{4d(1-\phi)} \left[ \hat{\theta}_1^2 + \hat{\theta}_2^2 - 2\phi \hat{\theta}_1 \hat{\theta}_2 \right].
\] (36)

Now, the difference between the terms in square brackets in (35) and (36) is

\[
\hat{\theta}_1(\hat{\theta}_1 - \theta_1) + \hat{\theta}_2(\hat{\theta}_2 - \theta_2) - \phi \hat{\theta}_1(\hat{\theta}_2 - \theta_2) - \phi(\hat{\theta}_1 - \theta_1),
\]

the expected value of which over θ and s is zero, since the expected value of each term \( \hat{\theta}_i(\hat{\theta}_j - \theta_j) \) with \( i, j \in \{1, 2\} \) can be shown to be zero. It follows that the expected value of the term in square brackets in (36) is again 2(1 − φ + k^2t^2η)̃θ^2. Collecting terms, the expected total surplus is \( \frac{\theta^2}{2d(1-\phi)}(1 - \phi + k^2t^2\eta) \). Finally, the social value of knowledge is the difference between this expression for the actual k and for k = 0, which is v as stated in the proposition.

2. Since η depends only on k, ρ and φ, v is obviously increasing in \( \bar{\theta} \) and t, decreasing in d, and independent of e. Next, v is increasing in k if and only if \( k^2\eta \) is increasing in k. That is the case, since substituting from (8) one can compute

\[
\frac{\partial (k^2\eta)}{\partial k} = \frac{2k[(1-\rho)^2(1+k^2\rho)^2 + 2\rho(1-\phi)(1-k^2)(1-k^2\rho^2)]}{(1-k^4\rho^2)^2} > 0.
\]

Next, v decreasing in ρ if and only if η is decreasing in ρ. The derivative of η with respect to ρ is

\[
-\frac{k^2\phi(1-\rho^2)(1-k^4\rho^2) + 2(1-k^2)[\phi(1-k^2\rho^2) - \rho(1-k^2)]}{(1-k^4\rho^2)^2},
\]

which is negative if the numerator (i.e. the expression in (13)), is positive. The l.h.s. of (13) equals \( \phi(2-k^2) > 0 \) for \( \rho = 0 \), equals \(-2(1-\phi)(1-k^2)^2 < 0 \) for \( \rho = 1 \), and its derivative of the l.h.s. of (13) with respect to ρ is

\[
-2[(1-k^2)^2 + \phi \rho k^2(3-2k^2 + k^4(1-2\rho^2))],
\]

which is always negative. This establishes the existence of critical value \( \bar{\rho} \in (0, 1) \) such that (13) holds for all \( \rho \leq \bar{\rho} \). To establish the sufficient condition mentioned the proposition, notice that \( a_1(1,-1) > a_1(1,1) \) holds if and only if (33) is positive. Subtracting 2(1 − k^2) times (33) from (13) yields \( k^2\phi(1-\rho^2)(1-k^4\rho^2) \), which is positive. This means that whenever \( a_1(1,-1) > a_1(1,1) \), (13) also holds, implying that v is decreasing in ρ.

Finally, v is increasing in φ if and only if \( \eta/(1-\phi) \) is increasing in ρ. This is the case, since (using (8)) the derivative of η/(1 − φ) with respect to ρ is \( (1-\rho)^2/[(1-\phi)^2(1-k^2\rho)] \).
Proof of Proposition 5: Parts (a) and (b): for $k, \theta, t, d, \phi$ and $\rho$ the results follow immediately from Proposition 4 and (14). The positive effect of $e$ on $\beta^*$ follows from inspection of (9). The derivative of $\gamma^*$ with respect to $e$ is
\[
\frac{1}{2(1-e)^2} - \frac{d(1+e)(1-\phi)}{4(1-e)^3k^2t^2\eta\theta^2},
\]
which is negative if and only if
\[
d(1+e)(1-\phi) - 2(1-e)k^2t^2\eta\theta > 0.
\]
Next, use (7) and (29) to obtain the (ex-ante) expected value of $y$ as function of $\beta$ and $\gamma$:
\[
E(y) = \frac{e}{2} + (1-e) \left\{ \frac{\bar{\theta} \beta}{d} + \frac{(1-e)\gamma^2}{d(1-\phi)}(1-\phi + k^2t^2\eta) \right\}.
\]
The derivative of $E(y)$, evaluated at $\beta^*$ and $\gamma^*$ has the same sign as
\[
d(1-\phi)[e(1-\phi) + 2\eta k^2t^2] - 4(1-e)k^2t^2\eta(1-\phi + k^2t^2\eta)\bar{\theta}^2.
\]
The difference $2(1-\phi + k^2t^2\eta)$ times (37) minus (39) is
\[
d(1-\phi)[2(1-\phi) + e(1-\phi + 2k^2t^2\eta)] > 0,
\]
which means that $\gamma^*$ is decreasing in $e$ whenever $E(y)$ is increasing in $e$, proving part (a) for $e$.

Part (c): Except for $d$ and $e$, the profit (15) depends on the model’s parameters only through $v$; we can ignore any effects on $\beta^*$ or $\gamma^*$ because of the envelope theorem. The derivative of (15) with respect to $v$ is positive since $(1-e)\gamma^* < 1$. The results for $k, \theta, t, \phi$ and $\rho$ then follow from Proposition 4 and (15). For $d$, we first have that $v$ is decreasing in $d$. Moreover, the partial derivative of (15) with respect to $d$ has the same sign as $-\bar{\theta} - \beta^* - (1-e)\gamma^*\bar{\theta}$, which upon substituting for $\beta^*$ and $\gamma^*$ from (9) reduces to $-\bar{\theta}/2$. Hence both the direct and the indirect (through $v$) effect of $d$ on the profit are negative. For $e$, differentiate (30) with respect to $e$ and plug in $\beta^*$ and $\gamma^*$, to obtain an expression that is negative if and only if (A1) holds.  

$\blacksquare$
References


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Figure 1: Illustration of production, informational assumptions and performance measurement for a standard single-task agency model (panel a) and the scenario considered in this paper (panel b).